Isotropisation in the Approach to a Singularity

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Copernicus Webinar Series
October 25th, 2021

Mainly based on
Ganguly & JQ [arXiv:2109.11701]

Motivation

- The early universe can be explained by a period of inflation after the big bang
  $\longrightarrow$ produces a **flat**, **isotropic** universe with **scale-invariant** curvature perturbations

- However, other alternatives exist that could also explain the early universe
  $\longrightarrow$ often involves physics 'before the big bang'

- E.g., bouncing cosmology, where the 'primordial physics' occurs during a **contracting phase** (prior to a bounce and the onset of standard big bang cosmology with radiation domination)

Bouncing cosmology

✓ Flatness problem: $\Omega = 1$, where ($8\pi G_{\rm N} = 1$ throughout this talk)

$$\Omega \equiv \frac{\rho_{\rm matter}}{3H^2} = 1 + \frac{k}{(aH)^2}\,,$$

is an attractor for $\dot{a} < 0$ and $1 + 3w > 0$ ($w \equiv p/\rho$):

$$\text{Friedmann eqs.} \implies \frac{\mathrm{d}|\Omega - 1|}{\mathrm{d}t} = (1+3w)\left(\frac{\dot{a}}{a}\right)\Omega(\Omega - 1)$$

✓ Horizon problem: comoving horizon $|aH|^{-1}$ is very large initially and shrinks

✓ Structure formation problem: certain fields can generate scale-invariant scalar perturbations, e.g.,

- adiabatic **'dust field'** Wands [gr-qc/9809062], Finelli & Brandenberger [hep-th/0112249]
- entropic **negative exponential scalar field** (ekpyrotic) Lehners *et al.* [hep-th/0702153], Buchbinder *et al.* [hep-th/0702154]

Bouncing cosmology

✗ Anisotropy problem: anisotropies typically tend to grow and dominate over everything else as $a \searrow 0$ (in the approach to the would-be big crunch singularity)

For different components with $\rho^{(w)} \propto a^{-3(1+w)}$ $(w < 1)$,

$$3H^2 = \Lambda - \frac{k}{a^2} + \frac{\rho_0^{(\mathrm{mat.})}}{a^3} + \frac{\rho_0^{(\mathrm{rad.})}}{a^4} + \frac{\rho_0^{(\mathrm{ani.})}}{a^6} \,.$$

(This does not appear to be a problem for matter with $w > 1$.)

$\Rightarrow$ Any bouncing alternative to inflation that wants to be viable and legitimately considered must solve these problems:

- anisotropies must not disrupt the contracting background that generates the right perturbations;
- anisotropies must not disrupt the bounce.

Anisotropy evolution in GR with a perfect fluid

- Consider a Bianchi type-I metric:

$$g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -\mathrm{d}t^2 + a(t)^2\sum_{i=1}^{3} e^{2\beta_{(i)}(t)}(\mathrm{d}x^i)^2\,, \quad \sum_{i=1}^{3}\beta_{(i)}(t) = 0$$

$$a_{(i)} = ae^{\beta_{(i)}}\,, \quad \ln a = \langle \ln a_{(i)}\rangle\,, \quad H \equiv \dot{a}/a = \langle H_{(i)}\rangle\,, \quad H_{(i)} = H + \dot{\beta}_{(i)}$$

- Hypersurface with timelike unit normal u^μ has

$$\underbrace{K_{\mu\nu}}_{\text{extr. curv.}} = (\underbrace{g_{\mu\nu} + u_\mu u_\nu}_{h_{\mu\nu}})H + \underbrace{\sigma_{\mu\nu}}_{\text{shear}}\,, \quad {\sigma_i}^j = \dot{\beta}_{(i)}{\delta_i}^j\,, \quad \sigma^2 \equiv \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\sum_{i=1}^{3}\dot{\beta}_{(i)}^2$$

- Einstein field equations $\implies$ FLRW + shear anisotropy component:

$$3H^2 = \rho + \sigma^2\,, \quad 2\dot{H} = -(\rho + p) - 2\sigma^2$$

$$\dot{\sigma}_i{}^j + 3H{\sigma_i}^j = 0 \implies \ddot{\beta}_{(i)} + 3H\dot{\beta}_{(i)} = 0 \implies \dot{\beta}_{(i)} \propto a^{-3}$$

$$\implies p_\sigma = \rho_\sigma = \sigma^2 = \frac{1}{2}\sum_{i=1}^{3}\dot{\beta}_{(i)}^2 \propto a^{-6}$$

Anisotropy evolution in GR with a perfect fluid

- Shear anisotropy component behaves as a set of massless scalar fields $\mathcal{L} = -\frac{1}{2}\partial_\mu\beta_I\partial^\mu\beta^I$, i.e., with stiff EoS

- When the shear anisotropy component dominates,

$$H^2 \sim \sigma^2 \propto a^{-6} \implies a(t) \sim |t|^{1/3}$$

$\longrightarrow$ Kasner singularity as $t \nearrow 0$
$\longrightarrow$ 'Belinski-Khalatnikov-Lifshitz (BKL) instability'

- Tuning the initial conditions for anisotropies to remain subdominant would be quite huge (see additional slides)

Ekpyrosis

- How about scalar fields with negative exponential potential?

$$\mathcal{L} = \sum_I \left(-\frac{1}{2}\partial_\mu \phi_I \partial^\mu \phi_I + V_I e^{-c_I \phi_I} \right) , \quad V_I > 0 \, , \; c_I^2 > 6$$

- Those can arise as moduli of higher-dim. brane constructions in string theory (e.g., distance between 'end-of-the-world' branes)
- Background scaling solution:

$$a(t) \propto (-t)^{1/\epsilon} , \quad \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2} \left(\sum_I c_I^{-2} \right)^{-1} > 3 \, , \quad w = \frac{2\epsilon}{3} - 1 > 1$$

- Direction $\perp$ to background trajectory in field space generates scale-invariant scalar perturbations

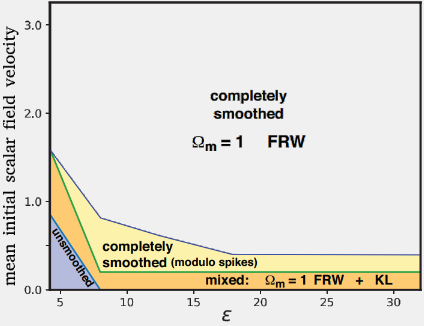
Isotropisation in ekpyrosis

- Ekpyrotic fields dilute anisotropies

$$3H^2 = \ldots + \frac{\rho_0^{(\mathrm{ani.})}}{a^6} + \frac{\rho_0^{(\mathrm{ek.})}}{a^{2\epsilon}}\,, \qquad \epsilon > 3$$

$\longrightarrow$ FRW is an attractor "Cosmic no hair for collapsing universes", Lidsey [hep-th/0511174]

- Highly efficient and even robust to large anisotropic, curved, and inhomogeneous ICs Garfinkle *et al.* [0808.0542], figure below addapted from Ijjas *et al.* [2006.04999]



Isotropisation with massive gravity

- In GR:

$$S \supset \int \mathrm{d}^3x\mathrm{d}t\, a^3 \left(\frac{1}{2}\dot{\beta}_{(i)}^2\right) \quad \overset{\delta_{\beta_{(i)}}S=0}{\Longrightarrow} \quad \ddot{\beta}_{(i)} + 3H\dot{\beta}_{(i)} = 0$$

- But if the graviton has a mass m_g:

$$S \supset \int \mathrm{d}^3x\mathrm{d}t\, a^3 \left(\frac{1}{2}\dot{\beta}_{(i)}^2 - \frac{1}{2}m_g^2\beta_{(i)}^2\right) \overset{\delta_{\beta_{(i)}}S=0}{\Longrightarrow} \ddot{\beta}_{(i)} + 3H\dot{\beta}_{(i)} + m_g^2\beta_{(i)} = 0$$

- If $m_g^2 \gg H^2$, then anisotropies behave like an oscillating massive field with matter EoS in average: the EOMs are solved for

$$\beta_{(i)}(t) \propto \frac{\sin(m_g t)}{m_g t}\,, \quad H(t) = \frac{2}{3t} \implies \rho_\sigma = \frac{1}{2}\sum_{i=1}^{3}\left(\dot{\beta}_{(i)}^2 + m_g^2\beta_{(i)}^2\right) \propto a^{-3}$$

⇒ Anisotropies may be subdominant even during matter domination!

⇒ Solves many issues of “matter bounce cosmology” at once

Lin, Brandenberger & JQ [1711.10472]

Isotropisation with massive gravity

- 0-mode tensor perturbations $\equiv$ anisotropies:

$$\delta g_{ij} = a^2\gamma_{ij} \implies \sigma_i{}^j = \frac{1}{2}\dot{\gamma}_i{}^j$$

$$\implies S \supset \int \mathrm{d}^3x\mathrm{d}t\, a^3 \left((\dot{\gamma}_i{}^j)^2 - (\vec{\nabla}\gamma_i{}^j)^2 - m_g^2(\gamma_i{}^j)^2\right)$$

$$\overset{\vec{\nabla}\gamma_i{}^j\to 0}{\implies} \quad \ddot{\gamma}_i{}^j + 3H\dot{\gamma}_i{}^j + m_g^2\gamma_i{}^j = 0 \quad \longrightarrow \gamma_i{}^j \text{ suppressed}$$

$\Rightarrow$ Solves the large tensor-to-scalar ratio problem of matter bounce cosmology JQ *et al.* [1508.04141], Li, JQ *et al.* [1612.02036]

- $m_g \lesssim \mathcal{O}(10^{-23}\,\mathrm{eV})$ today $\implies m_g$ would have had to be time dependent to have $m_g > |H_{\mathrm{bounce}}|$
- Adding a mass to a spin-2 field typically excites 3 new d.o.f.
- Only the 2 standard polarisation modes if Lorentz invariance is partially broken Dubovsky *et al.* [hep-th/0411158], Lin & Labun [1501.07160], Lin & Sasaki [1504.01373], Domènech *et al.* [1701.05554], Lin & Mukohyama [1708.03757], Kuroyanagi *et al.* [1710.06789]

Isotropisation with a non-perfect fluid

Details in Ganguly & JQ [arXiv:2109.11701] from here on

- Ignoring heat transfer,

$$T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu} + \pi_{\mu\nu}$$

$\longrightarrow$ EOMs are modified:

$$\dot{\rho} + 3H(\rho + p) = -\pi^{ij}\sigma_{ij}\,, \qquad \dot{\sigma}_i{}^j + 3H\sigma_i{}^j = \pi_i{}^j$$

- A fluid with shear viscosity has an anisotropic stress according to

$$\pi_{ij} = -2\eta\sigma_{ij}$$

- AdS/CFT points to a universal lower bound on shear viscosity

  e.g., Son & Starinets [0704.0240]

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

- From kinetic theory,

$$\eta \sim c_{\rm s}\rho\ell_{\rm mfp}$$

Toy model: finite-temperature interacting field theory

- Canonical scalar field, minimally coupled to gravity, with potential

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

  Extreme regimes:
  - $T \ll m$, matter-like, $\rho \sim a^{-3}$
  - $T \gg m/\lambda$, radiation-like, $\rho \sim a^{-4} \propto T^4$

- At high-T, the $\lambda\phi^4$ self interaction implies a cross-section

$$\sigma \sim \frac{\lambda^2}{T^2} \implies \ell_{\mathrm{mfp}} \sim \frac{1}{n\sigma} \sim \frac{1}{\lambda^2 T} \implies \eta \sim \rho\ell_{\mathrm{mfp}} \sim \frac{T^3}{\lambda^2} \propto a^{-3}$$

- Note that $\eta/s \sim 1/\lambda^2 \gtrsim 1$ for $\lambda \lesssim 1$

Isotropisation with an interacting scalar field theory

- Shear evolution ($\eta = \kappa/a^3$, $\kappa > 0$):

$$\dot{\sigma}_i{}^j + 3H\sigma_i{}^j = \pi_i{}^j = -2\eta\sigma_i{}^j \implies \dot{\sigma}_i{}^j + 3H\sigma_i{}^j = -2\frac{\kappa}{a^3}\sigma_i{}^j$$

- Assuming radiation domination initially (FLRW), $a(t) = \sqrt{t/t_0}$ ($t, t_0 < 0$) and the solution reads

$$\sigma^2 \propto \frac{1}{a^6}\exp\left(-\frac{8\kappa|t_0|}{a}\right) \xrightarrow{a\searrow 0} 0$$

⊗ Caveat: one cannot trust this all the way to $a \searrow 0$ since viscosity only makes sense on length scales smaller than the size of the system, here when $\ell_{\mathrm{mfp}} < |H|^{-1}$, but $\ell_{\mathrm{mfp}} \sim T^{-1} \sim a$ and $H^2 \sim \rho \sim a^{-4}$

⊗ In the similar spirit, it does not make sense to take the $\lambda \searrow 0$ limit

Isotropisation with an interacting scalar field theory

- Consider a situation where shear is already dominating the universe

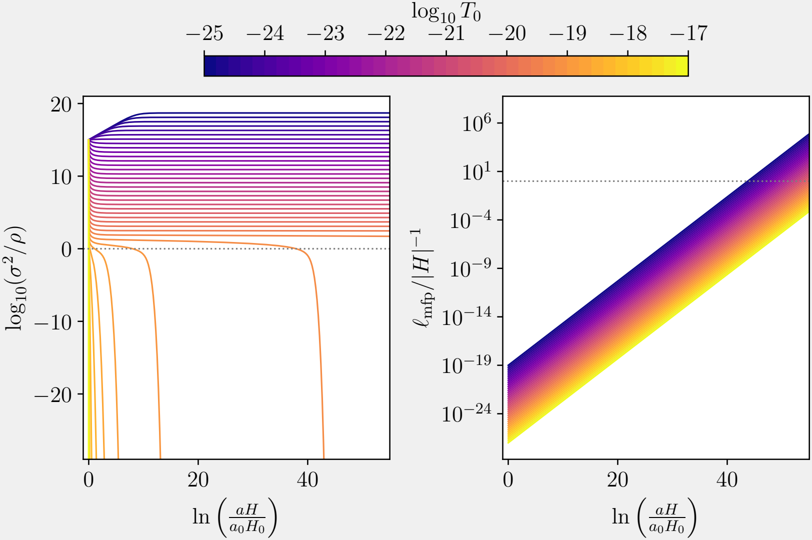
- Can the viscosity from the subdominant radiation-like interacting scalar field isotropise the universe while in the regime $\ell_{\mathrm{mfp}} < |H|^{-1}$?

- Let's numerically solve

$$\dot{\rho} + 4H\rho = \frac{4T_0^3}{\lambda^2}\left(\frac{a_0}{a}\right)^3\sigma^2\,, \qquad \dot{\sigma}_i{}^j + 3H\sigma_i{}^j = -\frac{2T_0^3}{\lambda^2}\left(\frac{a_0}{a}\right)^3\sigma_i{}^j$$

$$\lambda = 10^{-3}\,, \quad H_0 = -10^{-50}\,, \quad \frac{\sigma_0^2}{\rho_0} = 10^{15}$$

Isotropisation with an interacting scalar field theory



Isotropisation with an interacting scalar field theory

- Is this robust to the inclusion of curvature anisotropies?

▶ In a Bianchi type-IX spacetime,

$$h^i{}_j = a^2\,\mathrm{diag}\left(e^{2\beta_+ + 2\sqrt{3}\beta_-},\, e^{2\beta_+ - 2\sqrt{3}\beta_-},\, e^{-4\beta_+}\right)$$

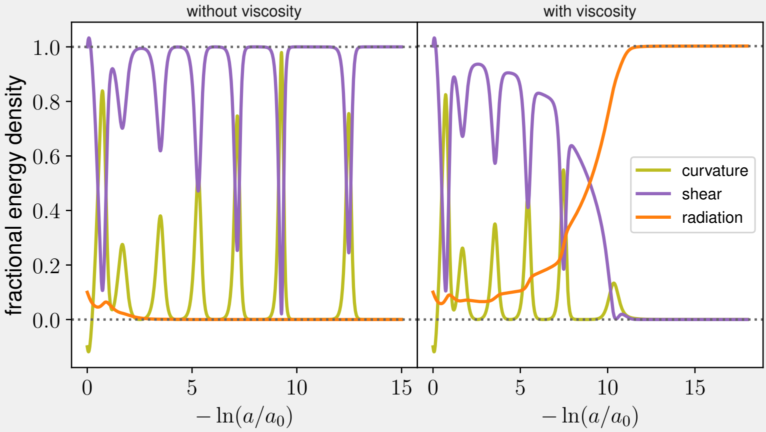
$$U(\beta_+, \beta_-) = \frac{1}{4}e^{-8\beta_+} - e^{-2\beta_+}\cosh\left(2\sqrt{3}\beta_-\right) + e^{4\beta_+}\sinh^2\left(2\sqrt{3}\beta_-\right)$$

$$3H^2 = \rho + \sigma^2 + \frac{1}{a^2}U(\beta_+, \beta_-)\,, \quad -2\dot{H} = \rho + p + 2\sigma^2 + \frac{2}{3a^2}U(\beta_+, \beta_-)$$

$$\dot{\rho} + 3H(\rho + p) = 4\eta\sigma^2\,, \quad \ddot{\beta}_\pm + 3H\dot{\beta}_\pm + \frac{1}{6a^2}\partial_{\beta_\pm}U = -2\eta\dot{\beta}_\pm\,, \quad \sigma^2 = 3(\dot{\beta}_+^2 + \dot{\beta}_-^2)$$

Isotropisation with an interacting scalar field theory

$$\frac{\rho_0}{3H_0^2} = \frac{1}{10}\,, \quad \frac{\sigma_0^2}{3H_0^2} = 1\,, \quad \frac{^{(3)}R_0}{6H_0^2} = \frac{1}{10}$$



What about other cosmic fluids exhibiting viscosity

- Consider a gas of nearly pressureless dust-like matter: this is typically unstable to gravitational collapse

  - Fluid with small sound speed $\implies$ gravitational instability $\implies$ black hole formation JQ & Brandenberger [1609.02556], Chen *et al.* [1609.02571]

  - Black holes attract each other gravitationally (they 'interact') $\implies$ a 'fluid of black holes' is viscous

  - Small black holes ($R \ll |H|^{-1}$), dilute gas:

$$\sigma \sim \left(\frac{R}{c_\mathrm{s}^2}\right)^2, \qquad \ell_\mathrm{mfp} \sim \frac{1}{n\sigma} \sim \frac{c_\mathrm{s}^4}{\rho R}, \qquad \eta \sim c_\mathrm{s}\rho\ell_\mathrm{mfp} \sim \frac{c_\mathrm{s}^5}{R} \approx \mathrm{const.}$$

$$\dot{\sigma}_i{}^j + 3H\sigma_i{}^j = -2\eta\sigma_i{}^j \implies \sigma^2 \sim \frac{e^{4\eta|t|}}{a^6}, \ |t| \searrow 0$$

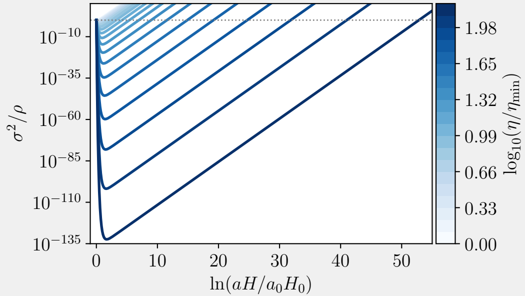
Dilute dust-like black hole gas

- If the background is matter dominated at first (FLRW),

$$\frac{\sigma^2}{\rho} = \left(\frac{a_0}{a}\right)^3 \exp\left[-\frac{4\eta}{3|H_0|}\left(1-\left(\frac{a}{a_0}\right)^{3/2}\right)\right]$$

- One has isotropisation only if σ^2/ρ is decreasing, which can happen if

$$\eta > \frac{3}{2}|H_0| \equiv \eta_{\rm min}$$



Dilute dust-like black hole gas

- The problem here is that $\eta > \eta_{\mathrm{min}}$ only if

$$c_{\mathrm{s}}^5 > R|H_0|\,,$$

but recall that viscosity only makes sense if $\ell_{\mathrm{mfp}} < |H|^{-1}$, which amounts to

$$c_{\mathrm{s}}^4 \lesssim R|H|\,.$$

✗ Therefore, a dilute gas of black holes cannot realistically be viscous enough to isotropise a dust-like contracting universe

Let's push the black hole gas picture to the limit

- Imagine large black holes ($R \sim |H|^{-1}$) that dominate the universe near a big crunch
- Conjectured to be the state of matter at high density in the early universe, e.g., in string theory Banks & Fischler [many papers], Veneziano [e.g., hep-th/0312182], Masoumi & Mathur [1406.5798], Masoumi [1505.06787], JQ, Brandenberger, Gasperini & Veneziano [1809.01658], Mathur [2009.09832]
- Consider a volume with $N \sim V/R^3$ black holes:

$$E \sim NM \sim \frac{V}{R^2}\,, \quad S \sim NR^2 \sim \frac{V}{R} \implies S \sim \sqrt{EV}$$

$$\implies T = \left(\frac{\partial S}{\partial E}\right)_V^{-1} \sim \sqrt{\frac{E}{V}} = \sqrt{\rho}\,, \quad p = T\left(\frac{\partial S}{\partial V}\right)_E = \frac{E}{V} = \rho$$

- Stiff $p = \rho$ fluid with $s \equiv S/V \sim \sqrt{\rho}$
- No shear by construction since we recover the Friedmann equation

$$\rho = \frac{E}{V} \sim \frac{1}{R^2} \sim H^2$$

Dense black hole gas viscosity

- As before

$$\sigma \sim R^2 \,, \quad \ell_{\mathrm{mfp}} \sim \frac{1}{n\sigma} \sim R \,, \quad \eta \sim \frac{1}{R} \sim |H| \sim \sqrt{\rho} \,, \quad \frac{\eta}{s} = \mathrm{const.}$$

- ▶ So for $\eta = \kappa |H|$ with κ of order 1, we have

$$\dot{\sigma}_i{}^j + 3H\sigma_i{}^j = 2\kappa H \sigma_i{}^j \implies \sigma^2 \propto \frac{1}{a^{6-4\kappa}}$$

$\Rightarrow$ forbids anisotropies from winning over compared to the ‘stiff background’ with $\rho \propto a^{-6}$:

$$\frac{\sigma^2}{\rho} \propto a^{4\kappa} \xrightarrow{a \searrow 0} 0$$

- In other words, anisotropies can never develop
- Only microphysical example of $\eta \propto \sqrt{\rho}$, which was known as a parametrisation to lead to an isotropic singularity (full isotropisation by the time $a = 0$) Belinski [1310.5112], Belinski & Henneaux, Ganguly & Bruni [1902.06356], Ganguly [2008.02286]

Summary

- In GR with a $T_{\mu\nu}$ satisfying the DEC, anisotropies always end up dominating in a contracting universe
  $\longrightarrow$ it's a problem for bouncing cosmology
  $\longrightarrow$ e.g., it would involve fine tuning the ICs in matter domination more than to resolve the curvature problem of standard big bang cosmology (see additional slides)

- Ekpyrosis ($p_\phi > \rho_\phi$) appears well suited and robust as a resolution to this problem

- Other resolutions exist though:
  - massive gravity
  - viscous fluid (any realistic interacting fluid)
  - other modified gravity (e.g., limiting curvature Sakakihara, Yoshida, Takahashi & JQ [2005.10844]; see additional slides)

Summary

Isotropisation due to viscosity:

- Finite-T $\lambda\phi^4$ theory can robustly isotropise the universe to radiation-dominated FLRW for $\mathcal{O}(100)$ e-folds, but not all the way to arbitrarily small a
  $\longrightarrow$ could still be part of a bigger scenario
- Dust-like fluid as a dilute gas of black holes exhibits viscosity, but not enough to remain isotropic
  $\longrightarrow$ still BKL unstable, so no good for matter bounce
- Hypothetical dense black hole gas is viscous ($\eta \propto \sqrt{\rho}$) and is robust against the growth of anisotropies
  $\longrightarrow$ relevant for stringy constructions proposing this state of matter

$\Rightarrow$ Realistic fluids have interactions at the microscopic level and therefore viscosity, which can often play an important role in the cosmology
$\longrightarrow$ e.g., gravitational wave damping

Thank you for your attention!

I acknowledge support from the following agencies:

Fonds de recherche Nature et technologies Québec

NSERC CRSNG

Additional slides

How much of a problem?

Can't we fine-tune the initial conditions? Levy [1611.08972]

- Recall

$$\Omega_k \equiv \frac{\rho^{(\mathrm{curv.})}}{3H^2} \propto \frac{1}{(aH)^2} = \frac{1}{\dot{a}^2}$$

- Flatness problem in standard big bang cosmology (say radiation dominated with $a \propto \sqrt{t}$, $T \propto 1/a$):

$$\frac{\Omega_k(t_{\mathrm{today}})}{\Omega_k(t_{\mathrm{Pl}})} = \frac{t_{\mathrm{today}}}{t_{\mathrm{Pl}}} = \left(\frac{T_{\mathrm{Pl}}}{T_{\mathrm{today}}}\right)^2 \approx e^{146}$$

$\longrightarrow$ Exaggerated here, but that's why we typically say that we need 60 e-folds of inflation, where

$$\mathcal{N} \propto \ln(a|H|)$$

- Say we want 60 e-folds of matter-dominated contraction now with

$$a \propto |t|^{2/3} \implies |H| \propto a^{-3/2}$$

$$\implies \mathcal{N} \equiv \ln\left(\frac{a|H|}{a_0|H_0|}\right) = \frac{1}{2}\ln\left(\frac{a_0}{a}\right)$$

- Anisotropy problem ($f \equiv \rho^{(\mathrm{ani.})}/\rho^{(\mathrm{mat.})}$):

$$\frac{f}{f_0} = \left(\frac{a}{a_0}\right)^{-3} = e^{6\mathcal{N}} \stackrel{!}{=} e^{360}$$

- Big fine-tuning problem. Way more than the flatness problem of standard big bang cosmology

⇒ Very hard to get 'stable' matter-dominated contraction

Limiting (extrinsic) curvature

$$\mathcal{L} \supset \chi\mathcal{I} - V(\chi) \overset{\delta_\chi S=0}{\Longrightarrow} \mathcal{I} = V'(\chi) \qquad [\text{here } \mathcal{I} = \mathcal{I}(K_{\mu\nu}, h_{\mu\nu}, \mathrm{D}_\mu)]$$

$$|V'(\chi)| \leq \text{const.} \implies |\mathcal{I}| \leq \text{const.}$$

- Can we construct $\mathcal{I} \propto \sigma^2$ in Bianchi I? Sure:

$$\mathcal{I} \equiv K^\mu{}_\nu K^\nu{}_\mu - \frac{1}{3}(K^\mu{}_\mu)^2 \overset{\text{BI}}{=} 6\sigma^2$$

- Then, introduce a new vector field A_μ with

$$\mathcal{L} \supset \lambda(A_\mu A^\mu + 1) \overset{\delta_\lambda S=0}{\Longrightarrow} A_\mu A^\mu = -1$$

$\longrightarrow$ defines a hypersurface with normal unit timelike vector $n^\mu = A^\mu$, so then $\mathcal{I} = \nabla^\mu A_\nu \nabla^\nu A_\mu - \frac{1}{3}(\nabla^\mu A_\mu)^2$ and the whole theory has

$$\mathcal{L} \supset \frac{R}{2} + \lambda(A_\mu A^\mu + 1) + \chi\left(\nabla^\mu A_\nu \nabla^\nu A_\mu - \frac{1}{3}(\nabla^\mu A_\mu)^2\right) - V(\chi)$$

$\longrightarrow$ generalised mimetic/cuscuton/æther gravity in which shear anisotropies cannot blow up! Sakakihara, Yoshida, Takahashi & JQ [2005.10844]