Discriminating Between Theories of the Very Early Universe

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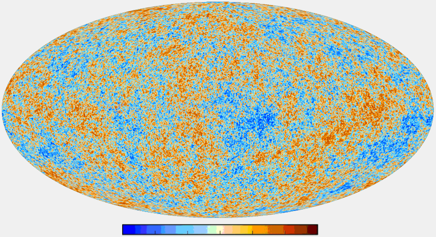
What we know of the very early universe with certainty

Figures adapted from Planck [arXiv:1502.01582,1807.06211]

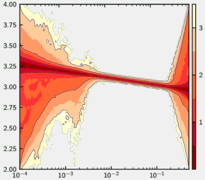
Cosmic Microwave Background $\sim 300{,}000$ yr after the 'big bang'



temperature fluctuations $\sim 10^{-4}$

primordial power spectrum $\sim 10^{-32}$ sec after the 'big bang'
(variance of spacetime curvature perturbations)

$\log\langle\zeta_k\zeta_k\rangle'$



standard deviation

Fourier wavenumber k [Mpc^{-1}]

$$\langle\zeta_k\zeta_k\rangle' = \underbrace{(2.10 \pm 0.03) \times 10^{-9}}_{A_s = \text{amplitude}} \left(\frac{k}{0.05\,\mathrm{Mpc}^{-1}}\right)^{\overbrace{-0.0351 \pm 0.0042}^{n_s - 1 = \text{tilt}}}$$

$\Rightarrow$ Nearly scale-invariant, Gaussian, scalar fluctuations

$\Rightarrow$ Currently no (statistically significant) sign of anything else!
(e.g., primordial gravitational waves, non-Gaussianities, running of the spectrum, features, etc.)

$\Rightarrow$ Incredibly rich and complex, yet very simple!

How can this be explained?

ASSUME initial conditions: quantum vacuum on small scales $\xrightarrow[?\ ?\ ?]{\text{Evolve}}$ MEASURE 'final conditions': $\langle\zeta_k\zeta_k\rangle' \sim k^0$ on large scales

linearized Einstein equations $\Rightarrow$

$$\partial_\tau^2(z\zeta_k) + \left(c_\mathrm{s}^2k^2 - \frac{\partial_\tau^2 z}{z}\right) z\zeta_k = 0\,, \quad z \equiv \frac{a\sqrt{2\epsilon}}{c_\mathrm{s}}$$

$\tau =$ conformal time

$a(\tau) =$ scale factor of the universe

$\epsilon(\tau) =$ characterizes the equation of state of the matter content

$c_\mathrm{s}(\tau) =$ sound speed

How can this be explained?

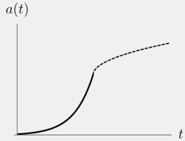
- E.g., $c_{\mathrm{s}} = 1$, $\epsilon \approx \mathrm{const.}$, $a(\tau) \sim |\tau|^{1/(\epsilon-1)} \quad \Rightarrow \quad \partial_\tau^2 z/z = \frac{2-\epsilon}{(\epsilon-1)^2\tau^2}$
  Approximate scale invariance is found for $\partial_\tau^2 z/z \approx 2/\tau^2$
  $\Rightarrow$ Inflation: $\epsilon \ll 1$ (negative pressure, approx. vacuum EoS)
  $\Rightarrow$ Fast contraction: $\epsilon \approx 3/2$ (pressureless matter) Wands [gr-qc/9809062]

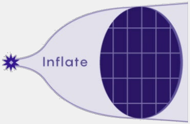
- Time-dependent ϵ or c_{s} or additional fields open up many more possibilities
  e.g., Hinterbichler & Khoury [1106.1428], Geshnizjani *et al.* [1107.1241]
  E.g.:
  - Slow contraction (a.k.a. ekpyrosis): $\epsilon > 3$ (ultra-stiff EoS)
    e.g., Lehners *et al.* [hep-th/0702153]
  - Slow expansion (a.k.a. genesis): $\epsilon < 0$ (ghost-like EoS)
    e.g., Creminelli *et al.* [1007.0027]

- **Can all be made consistent with the measured $\langle\zeta_k\zeta_k\rangle'$**

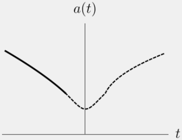
- More scenarios are also possible (e.g., 'beyond semi-classical GR'), but let's keep it simple for today

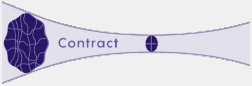
- Inflation (standard paradigm)



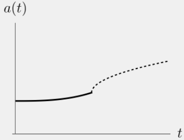


- Contraction (alternative)





- Slow expansion (alternative)



Can we realistically tell them apart?

With the running of the spectral index $\alpha_{\rm s}$, non-Gaussianities ($f_{\rm NL}$, $g_{\rm NL}$, ...), tensor-to-scalar ratio r, tensor tilt $n_{\rm t}$, etc.?

Surely. But one can often find models that lead to closely degenerate predictions.

Example:

single-field slow-roll inflation

$$n_{\rm s} \approx 0.97\,,\ \alpha_{\rm s} \approx -5\times 10^{-4}$$

$$f_{\rm NL}^{\rm local} \approx 0.01$$

$$r \approx 0.01\,,\ n_{\rm t} \approx -0.001$$

two-field ekpyrosis (slow contraction)

$$n_{\rm s} \approx 0.97\,,\ \log_{10}(-\alpha_{\rm s}) \lesssim -2$$

$$f_{\rm NL}^{\rm local} \approx \frac{3}{2}\kappa_3\sqrt{\epsilon} + 5 \qquad (\in [-5,5])$$

$$r \lesssim 0.06\,,\ n_{\rm t} \gtrsim 0.12$$

Ijjas *et al.* [1404.1265], Lehners & Wilson-Ewing [1507.08112],

Fertig *et al.* [1607.05663],

Ben-Dayan+ [1604.07899,1812.06970] $\rightarrow$ sourced perturbations

from gauge field production

So are there ways of **discriminating** between those theories, in a **model-independent** way, both **theoretically and observationally**?

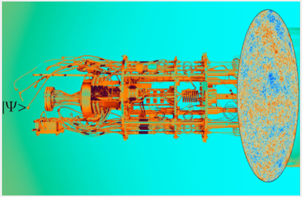
We need to invent new approaches!

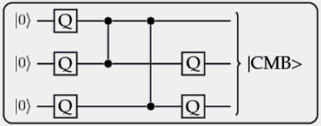
Let me propose a few avenues in that direction for the rest of this talk:

(1) Primordial quantum circuit complexity
(2) Primordial quantum transition amplitudes
(3) Primordial standard clocks

(1) Primordial quantum complexity

- How **complex** are the various scenarios? If we did a quantum simulation of the early universe, **how many quantum gates** would it require?





$$|\Psi_{\mathrm{Ref}}\rangle \xrightarrow{|\Psi_{\mathrm{Target}}\rangle=\hat{U}|\Psi_{\mathrm{Ref}}\rangle} |\Psi_{\mathrm{Target}}\rangle$$

- How many elementary quantum gates to construct $\hat{U}$? $\implies$ complexity
- The general idea is that a circuit can have a continuous differential-geometry description
  $\Rightarrow$ optimal quantum simulation $\equiv$ smallest number of gates $\equiv$ geodesic in the geometry of quantum gates
  Nielsen [quant-ph/0502070], Jefferson & Myers [1707.08570], Camargo *et al.* [1807.07075], Chapman *et al.* [1810.05151], Bhattacharyya+ [1810.02734,2001.08664,2005.10854], Lehners & JQ [2012.04911]

- Start with a Reference and a Target state, both Gaussian and with respective frequencies ω and Ω (1-d harmonic oscillators with 'position' ζ):

$$|\Psi_{\mathrm{R}}\rangle = \left(\frac{\omega}{\pi}\right)^{1/4} e^{-\frac{1}{2}\omega\zeta^2}\,, \quad |\Psi_{\mathrm{T}}\rangle = \left(\frac{\Omega}{\pi}\right)^{1/4} e^{-\frac{1}{2}\Omega\zeta^2}$$

- Example of gate that could constitute the unitary evolution ($\hat{\Pi} = -i\hat{\partial}_\zeta$):

$$\hat{Q} \equiv e^{\frac{\epsilon}{2}} e^{i\epsilon\hat{\zeta}\hat{\Pi}}\,, \qquad \hat{Q}|\Psi(\zeta)\rangle = e^{\frac{\epsilon}{2}}|\Psi(e^{\epsilon}\zeta)\rangle$$

- Then $\hat{U} = \hat{Q}^{\alpha}$ yields $|\Psi_{\mathrm{T}}\rangle = \hat{U}|\Psi_{\mathrm{R}}\rangle$ as long as $2\epsilon\alpha = \ln(\Omega/\omega)$
- Therefore, the # of gates (the complexity) goes as

$$\mathcal{C} = \epsilon\alpha = \frac{1}{2}\ln\left(\frac{\Omega}{\omega}\right) \xrightarrow{\omega,\Omega\in\mathbb{C}} \frac{1}{2}\left|\ln\left(\frac{\Omega}{\omega}\right)\right|$$

- In cosmology, $|\Psi_{\mathrm{R}}\rangle$ is the Bunch-Davies vacuum, and the late-time correlator is Lehners & JQ [2012.04911]

$$\Omega = z^2\left(-i\frac{\partial_\tau(z\zeta)^*}{(z\zeta)^*} + i\frac{\partial_\tau z}{z}\right)$$

Quantum circuit complexity

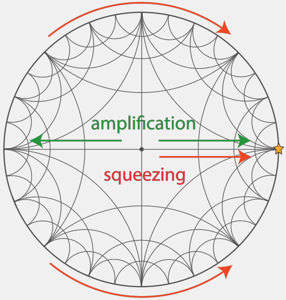
A convenient geometry is the hyperbolic one [it naturally arises when representing the Gaussian wavefunctions as covariance matrices, where elementary gates are elements of $\mathrm{Sp}(2,\mathbb{R})$] Camargo *et al.* [1807.07075]

Poincaré half-plane:

$$(x_0, y_0) = (0, 1) \longrightarrow (x, y) = \left(-\frac{\mathrm{Im}\,\Omega}{\sqrt{2}\,\mathrm{Re}\,\Omega}, \frac{\omega}{\mathrm{Re}\,\Omega}\right)$$

Poincaré disk:

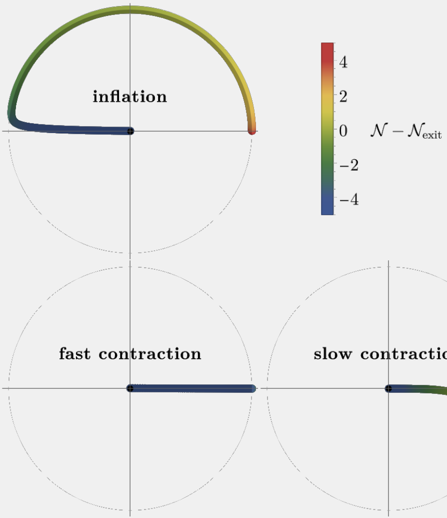
$$z = x + iy \longrightarrow \frac{z - i}{z + i}$$

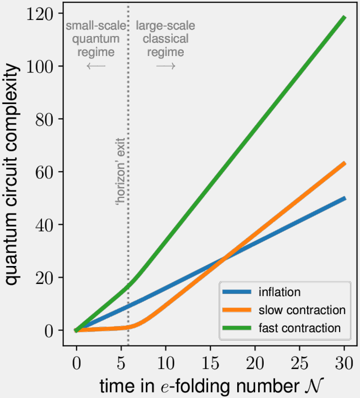


Lehners & JQ [2012.04911]

- **amplification** $\leftrightarrow 1/\mathrm{Re}\,\Omega \to \infty \leftrightarrow$ growth of $\langle \zeta_k \zeta_k \rangle'$
- **squeezing** $\leftrightarrow |\mathrm{Im}\,\Omega/\mathrm{Re}\,\Omega| \to \infty \leftrightarrow$ classicalization in the WKB sense
- **complexity** $\leftrightarrow$ hyperbolic distance from the origin

Complexity of early universe perturbations Lehners & JQ [2012.04911]





Complexity of early universe perturbations Lehners & JQ [2012.04911]

Super-horizon:

$$\text{inflation } (\epsilon < 1): \ \Delta\mathcal{C} \simeq \sqrt{2}\,\underbrace{(1+2\epsilon)}_{\approx 1}\Delta\mathcal{N}$$

$$\text{slow contraction } (\epsilon > 3): \ \Delta\mathcal{C} \simeq 2\sqrt{2}\,\underbrace{\left(\frac{\epsilon - 3/2}{\epsilon - 1}\right)}_{\approx 1}\Delta\mathcal{N}$$

$$\text{fast contraction } (\epsilon \approx 3/2): \ \Delta\mathcal{C} \simeq 3\sqrt{2}\,\Delta\mathcal{N}$$

$\Rightarrow$ inflation acts as a 'simple' quantum computer compared to its alternatives

$\Rightarrow$ very modest dependence on specific model realizations

✓ Good way to differentiate theories, theoretically speaking

▶ How can it be used to discriminate? Interpretation of chaos? Sensitivity to initial conditions?

(2) Primordial quantum amplitudes Jonas, Lehners & JQ [2012.04911]

$$\mathcal{A}(\Phi_\mathrm{i} \to \Phi_\mathrm{f}) = \int_{\Phi_\mathrm{i}}^{\Phi_\mathrm{f}} \mathcal{D}\Phi \, e^{\frac{i}{\hbar} S[\Phi]} \overset{\hbar \lll 1}{\simeq} \sum_{\text{saddles}} \mathcal{N} e^{\frac{i}{\hbar} S_{\text{on-shell}}[\Phi_\mathrm{i} \to \Phi_\mathrm{f}]}$$

$$\Phi = \{g_{\alpha\beta}, \phi, A_\mu, \ldots\}$$

$\rightarrow$ this only yields a well-defined (and non-zero) amplitude if the relevant saddle points have finite classical on-shell action

$$S_{\text{on-shell}}[\Phi_\mathrm{i} \to \Phi_\mathrm{f}] < \infty$$

(Off-shell contributions are expected to blow up, but this is completely fine quantum mechanically)

$\rightarrow$ E.g., in cosmology,

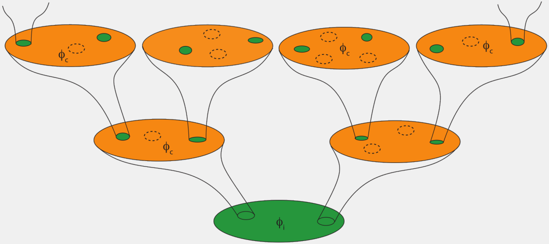
$$S_{\text{on-shell}} \sim \int_{t(\Phi_\mathrm{i})}^{t(\Phi_\mathrm{f})} \mathrm{d}t \, a\dot{a}^2 \overset{a \sim |t|^{1/\epsilon}}{\sim} \begin{cases} t^{\frac{3-\epsilon}{\epsilon}} \Big|_{0}^{t(\Phi_\mathrm{f})} & \text{inflation with } \epsilon \ll 1 \\ (-t)^{\frac{3-\epsilon}{\epsilon}} \Big|_{-\infty}^{t(\Phi_\mathrm{f})} & \text{contraction with } \epsilon > 1 \end{cases}$$

$\rightarrow$ Inflation appears to be fine, but contraction converges only if $\epsilon > 3$ (only slow contraction!)

But the story is not that simple for inflation

- If inflation really goes all the way back to the big bang singularity ($a = 0$), instabilities in the perturbations arise (interference among different saddle points) $\Rightarrow$ unviable
  Di Tucci, Feldbrugge, Lehners & Turok [1906.09007]

- If inflation is eternal (potential is so flat that field stochastically jumps up the potential and keeps inflating), action is divergent Jonas, Lehners & JQ [2102.05550]



$$S \sim \int_0^\infty \mathrm{d}t\, a^3\, V(\phi)\, \underbrace{\mathrm{Prob}[\phi \text{ is inflating at time } t]}_{\text{sol. to Fokker-Planck equation}} \to \infty \quad \text{if } \frac{|V_{,\phi}|}{V^{3/2}} < \frac{1}{\sqrt{2}\,\pi}$$

$\rightarrow$ Reminiscent of swampland criteria

e.g., Rudelius [1905.05198], Hamada, Montero, Vafa & Valenzuela [2111.00015]

Quadratic gravity (and beyond)

- Quadratic gravity is renormalizable Stelle [PRD 1977]

$$S_{\rm quad}=\int \mathrm{d}^4x\,\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R+\frac{\omega}{3\sigma}R^2-\frac{1}{2\sigma}C_{\mu\nu\rho\sigma}^2\right)$$

  $\rightarrow$ FLRW solutions $a(t)\sim t^s$ as $t\to 0^+$ lead to finite amplitudes only if $s>1$
  $\Rightarrow$ accelerating out of the big bang

  Lehners & Stelle [1909.01169], Jonas, Lehners & JQ [2102.05550]

  $\rightarrow$ In Bianchi I, only 'bounded anisotropy' solutions satisfy the principle
  e.g., constant-Hubble and constant-shear solution Barrow & Hervik [gr-qc/0610013]

- For some generic higher-curvature theory (up to Riem^n):

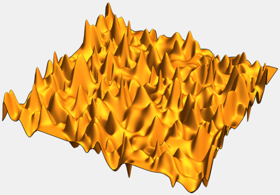
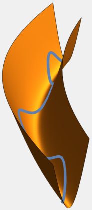
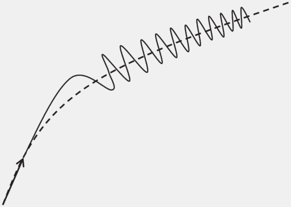
$$S_{\mathrm{Riem}^n}=\int \mathrm{d}^4x\,\sqrt{-g}\,f(R^\mu{}_{\nu\rho\sigma})$$

  $\rightarrow$ $a(t)\sim t^s$ solutions need to have $s>(2n-3)/3$
  $\Rightarrow$ If there are infinitely many ($n=\infty$), as potentially required, no such solutions respect the principle

✓ A finite cosmological amplitude principle is a good theoretical discriminator (but more model dependent)

(3) Primordial standard clocks

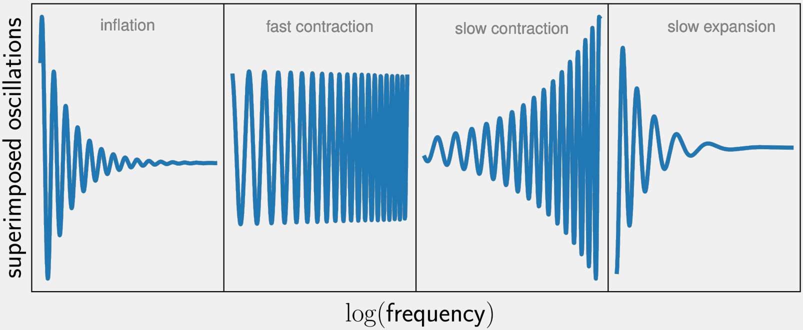
- One generally expects a wealth of heavy spectator fields in the early universe

- These oscillating heavy fields are expected to leave oscillatory signals in the observations
- And the frequency dependence is expected to mainly depend on the background evolution Chen+ [1104.1323,1106.1635,1404.1536,1411.2349,1601.06228,1608.01299]

Standard clocks

$$a(t) \sim |t|^{1/\epsilon} \longrightarrow \frac{\Delta\langle\zeta_k\zeta_k\rangle'}{\langle\zeta_k\zeta_k\rangle'_{\text{no oscil.}}} \sim k^{(\epsilon-3)/2} \sin\left(\frac{m/H_\star}{\epsilon(1-\epsilon)} k^\epsilon + \text{phase}\right)$$



$\rightarrow$ Oscillations superimposed on top of the nearly scale-invariant power spectrum **could tell us about ϵ and hence $a(t)$ in the very early universe!**

$\rightarrow$ Expected signals in other windows as well (3-pt function, GWs, etc.)

$\rightarrow$ Potentially observable with next generation of telescopes!

Chen+ [1605.09364,1605.09365,1610.06559,2106.07546]

$\rightarrow$ Explicit particle physics models have been constructed for inflation and the corresponding signals are currently extensively studied

Chen, Namjoo & Wang [1411.2349], Braglia *et al.* [2106.07546,2108.10110]

and not to mention the cosmological collider program (Arkani-Hamed & Maldacena [1503.08043], Lee, Baumann & Pimentel [1607.03735], Chen, Wang & Xianyu [1610.06597], etc.)

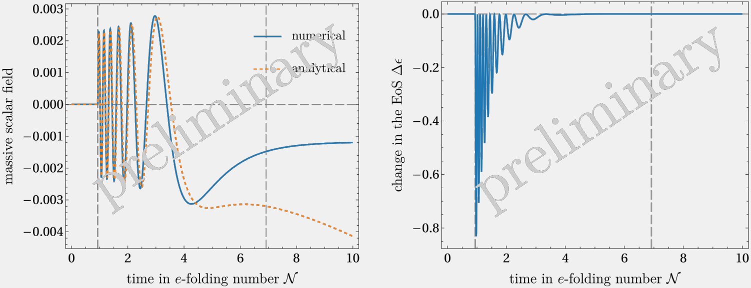
$\rightarrow$ Barely any exploration of the alternatives

First classical standard clock model in slow contraction

With Xingang Chen and Reza Ebadi

$$\mathcal{L} = \frac{R}{2} - \frac{1}{2}\mathcal{G}^{IJ}(\Phi_K)\partial_\mu\Phi_I\partial^\mu\Phi_J - V(\Phi_K)\,, \quad \Phi_K = (\phi, \chi, \sigma)$$

$$\ddot{\sigma} + 3H\dot{\sigma} + m^2\sigma = 0 \Rightarrow \sigma(t) \sim (-t)^{-3/(2\epsilon)}\sin(mt + \mathrm{phase})$$



$$\mathcal{L} \supset \sigma(\partial\chi)^2 \longrightarrow \mathcal{H}^{(2)}_{\mathrm{int}} \sim -a^3\sigma\left(\dot{\zeta}^2 - \frac{(\partial_i\zeta)^2}{a^2}\right) \Longrightarrow \frac{\Delta\langle\zeta_k\zeta_k\rangle'}{\langle\zeta_k\zeta_k\rangle'_{\text{no oscil.}}}$$

$\rightarrow$ **Exact signals currently under investigation, so stay tuned!**

Conclusions and future directions

- Very different realizations of the very early universe can degenerately predict the same simple nearly scale-invariant primordial spectrum
- We need new ways of discriminating between theories, in the most model-independent way:
  - $\rightarrow$ quantum circuit complexity:
    - ✓ nice description of the quantum-to-classical transition
    - ✓ very modest model dependence
    - applicability?
  - $\rightarrow$ finite quantum cosmological amplitudes:
    - ✓ strong theoretical constraint on allowed models
    - more model dependent
  - $\rightarrow$ standard clocks (heavy spectator fields):
    - ✓ strong potential observational constraints on allowed models
    - ✓ quite model independent
    - a lot more work to be done on the alternatives

Thank you for your attention!

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