Early Universe Cosmology in Fundamentally Motivated Alternative Theories of Gravity and the Principle of Finite Amplitudes

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Motivation

Barrow & Tipler's 1988 finite action principle (paraphrasing):

> the action S is the most basic quantity in physics,
> therefore it must be finite

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Action principles in nature

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Physical theories have their most fundamental expression as action integrals. This suggests that the total action of the Universe is the most fundamental physical quantity, and hence finite. In this article it is argued that finite universal action

Recently revisited by Barrow [arXiv:1912.12926]

$$S_{\text{Universe}} = \int_{\mathcal{M}_4} \mathrm{d}^4x\,\sqrt{-g}\,\mathcal{L}_{\text{all of gravity and matter}} < \infty$$

$\rightarrow$ otherwise claimed to be mathematically inconsistent to derive equations of motion

$\rightarrow$ integral must 'know' about both the past and the *future*

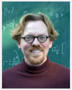
- But the action does play a fundamental role in the computation of *physical observables*, e.g.:

$$\mathcal{A}(\Phi_{\text{initial}} \to \Phi_{\text{final}}) = \langle \Phi_{\text{final}} | \Phi_{\text{initial}} \rangle = \int_{\Phi_{\text{initial}}}^{\Phi_{\text{final}}} \mathcal{D}\Phi\, e^{\frac{i}{\hbar} S[\Phi]}$$

$$\langle \prod_{i=1}^{n} \mathcal{O}_i \rangle = \int \mathcal{D}\Phi \left(\prod_{i=1}^{n} \mathcal{O}_i \right) e^{\frac{i}{\hbar} S[\Phi]}$$

$$\Phi = \{\phi, \psi_a, A_\mu, g_{\alpha\beta}, \ldots\}$$

Finite Amplitude Principle



Caroline Jonas, Jean-Luc Lehners & JQ [2102.05550]

The action that enters in the computation of any observable quantum amplitudes today should yield a non-zero, *finite*, well-defined result

$$\mathcal{A}(\Phi_{\mathrm{ini}} \to \Phi_{\mathrm{today}}) = \int_{\Phi_{\mathrm{ini}}}^{\Phi_{\mathrm{today}}} \mathcal{D}\Phi \, e^{\frac{i}{\hbar}S[\Phi]} \overset{\hbar \ll 1}{\simeq} \sum_{\mathrm{saddles}} \mathcal{N} e^{\frac{i}{\hbar}S_{\text{on-shell}}[\Phi_{\mathrm{ini}} \to \Phi_{\mathrm{today}}]}$$

- Very often, this translates into

$$S_{\text{on-shell}}[\Phi_{\mathrm{ini}} \to \Phi_{\mathrm{today}}] < \infty$$

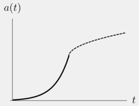
⋆ Off-shell contributions are expected to blow up, but that is completely fine quantum mechanically

Straightforwardly applicable to the early universe

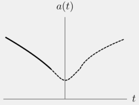
$$\text{GR+FLRW:}\quad a(t) \propto |t|^{1/\epsilon}\,,\quad \epsilon = \frac{3}{2}\left(1+\frac{p}{\rho}\right)$$

$$S_{\text{on-shell}} \sim \int_{t(\Phi_\mathrm{i})}^{t(\Phi_\mathrm{f})} \mathrm{d}t\, a\dot{a}^2 \sim \begin{cases} t^{\frac{3-\epsilon}{\epsilon}}\Big|_{0}^{t_\mathrm{f}} & \text{expansion with } \epsilon<3 \\ (-t)^{\frac{3-\epsilon}{\epsilon}}\Big|_{-\infty}^{t_\mathrm{f}} & \text{contraction with } \epsilon>1 \end{cases}$$

$\rightarrow$ Inflation ($\epsilon < 1$) appears to be fine



$\rightarrow$ Contraction converges only if $\epsilon > 3$ ($p > \rho$, i.e., slow contraction!)



- Only a semi-classical statement, but interesting to test when there are quantum effects, or for modified gravity and in alternative scenarios

Quadratic gravity (and beyond)

- Quadratic gravity is renormalisable Stelle [PRD 1977]

$$S_{\rm quad} = \int \mathrm{d}^4x\,\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R + \frac{\omega}{3\sigma}R^2 - \frac{1}{2\sigma}C_{\mu\nu\rho\sigma}^2\right)$$

  $\rightarrow$ FLRW solutions $a(t) \sim t^s$ as $t \to 0^+$ lead to finite amplitudes only if $s > 1$
  $\implies$ accelerating out of the big bang
  Lehners & Stelle [1909.01169]; Jonas, Lehners & JQ [2102.05550]

  $\rightarrow$ In Bianchi I, only 'bounded anisotropy' solutions satisfy the principle e.g., constant-Hubble and constant-shear solution Barrow & Hervik [gr-qc/0610013]

- For some generic higher-curvature theory (up to Riem^n):

$$S_{\mathrm{Riem}^n} = \int \mathrm{d}^4x\,\sqrt{-g}\,f(R^\mu{}_{\nu\rho\sigma})$$

  $\rightarrow$ $a(t) \sim t^s$ solutions need to have $s > (2n-3)/3$
  $\Rightarrow$ If there are infinitely many ($n = \infty$), as potentially required, no such solutions respect the principle

Limiting curvature

Constant-Hubble and constant-shear solutions can be found in quadratic gravity and within **limiting (extrinsic) curvature theory**
$\longrightarrow$ Original idea to bound curvature and avoid singularities Markov, Mukhanov, Frolov, Brandenberger, etc. [1980s, 1990s]; now see Sakakihara, Yoshida, Takahashi & JQ [2005.10844]

$$\mathcal{L} \supset \sum_a \chi_a \mathcal{I}_a(\mathbf{Riem}, \boldsymbol{g}, \boldsymbol{\nabla}) - V(\chi_1, \ldots, \chi_a)$$

$$\longrightarrow \sum_a \chi_a \mathcal{I}_a(\boldsymbol{K}, \boldsymbol{h}, \mathbf{D}) - V(\chi_1, \ldots, \chi_a)$$

$$\delta_{\chi_a} S = 0 \implies \mathcal{I}_a = \partial_{\chi_a} V$$

- Introduce unit timelike vector field u^μ with

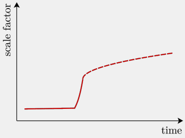
$$\mathcal{L} \supset \chi_1 \underbrace{(\nabla_\mu u^\mu)^2}_{\propto H^2} + \chi_2 \underbrace{\left[\nabla_\mu u^\nu \nabla_\nu u^\mu - \frac{1}{3}(\nabla_\mu u^\mu)^2\right]}_{\propto \sigma_{ij}\sigma^{ij} \propto \text{shear}} - V(\chi_1, \chi_2)$$

$$\delta_{\chi_{1,2}} S = 0 \implies H^2 \propto \partial_{\chi_1} V\,, \quad \sigma_{ij}\sigma^{ij} \propto \partial_{\chi_2} V \implies H^2 \text{ and } \sigma_{ij}\sigma^{ij} \text{ bounded}$$

$$\implies S_{\text{on-shell}} \sim \int_{-\infty} \mathrm{d}t\ \underbrace{a^3}_{\to 0} \times \underbrace{\{H^2, \dot{H}, \sigma_{ij}\sigma^{ij}, V\}}_{\to \text{constant}} < \infty$$

Stringy loitering

- A loitering (emerging) spacetime has $H, \dot{H} \to 0$ ($a \to \text{constant}$) as $t \to -\infty$



$$S_{\mathrm{EH}} \sim \int_{-\infty}^{t_0} \mathrm{d}t\, a^3(2H^2 + \dot{H}) < \infty$$

- e.g., string gas cosmology Brandenberger & Vafa [Nucl. Phys. B 1989] and genesis scenarios e.g., Creminelli, Nicolis & Trincherini [1007.0027]

- Actual implementations involve modified gravity (e.g., Horndeski) or string theory

- One recent development involves α' corrections to all orders Hohm & Zwiebach [1905.06963]; Bernardo, Brandenberger & Franzmann [2005.08324]; JQ, Bernardo & Franzmann [2105.01083]

$$S = \frac{1}{2\ell_{\mathrm{s}}^{d-1}} \int \mathrm{d}t\, a^d e^{-2\phi} \left(-4\dot{\phi}^2 + 4d\dot{\phi}H - d^2H^2 - 2d \sum_{k=1}^{\infty} (-\alpha')^{k-1} c_k 2^{2k} H^{2k} \right) + S_{\mathrm{matter}}$$

- Transformed to the Einstein frame:

$$S^{(\mathrm{EF})}=\int \mathrm{d}t_{\mathrm{E}}\, a_{\mathrm{E}}^{d}\left[\frac{3}{\kappa^2}\left(2H_{\mathrm{E}}^2+\frac{\mathrm{d}H_{\mathrm{E}}}{\mathrm{d}t_{\mathrm{E}}}\right)+\frac{1}{2}\left(\frac{\mathrm{d}\phi_{\mathrm{E}}}{\mathrm{d}t_{\mathrm{E}}}\right)^2+e^{2\kappa\phi_{\mathrm{E}}/\sqrt{d-1}}\mathcal{A}(H)\right]$$

$$\text{where}\qquad \mathcal{A}(H)=2d\sum_{k=2}^{\infty}(-\alpha')^{k-1}c_k 2^{2k}H^{2k}$$

- The string-frame solution has $H=\mathrm{constant}$, so the infinite polynomial $\mathcal{A}(H)=\mathrm{constant}$.
- Then the Einstein-frame solution has $H_{\mathrm{E}}=0$ and $\phi_{\mathrm{E}}\sim\ln(1/|t_{\mathrm{E}}|)$ as $t_{\mathrm{E}}\to-\infty\implies$

$$S^{(\mathrm{EF})}_{\text{on-shell}}\sim\int_{-\infty}\mathrm{d}t_{\mathrm{E}}\, a_{\mathrm{E}}^{d}\times 0\implies\text{converges}$$

Conclusions

- Idea that action plays a fundamental role allows us to put interesting constraints on theories

  Additional recent works in this direction: Borissova & Eichhorn [2012.08570]; Casadio, Kamenshchik & Kuntz [2102.10688]; Chojnacki & Kwapisz [2102.13556]; Giacchini, de Paula Netto & Modesto [2105.00300]

- Demanding a finite past cosmological amplitude allows (selected examples):
  - ✓ Transient and non-eternal inflationary phase within GR
  - ✓ Ekpyrotic cosmology (slow contraction)
  - ✓ No-boundary proposal
  - ✓ **Accelerating backgrounds in quadratic gravity**
  - ✓ **Limiting curvature constant Hubble and constant shear**
  - ✓ **Loitering phase in string cosmology (with α' corrections)**

  but rules out:
  - ✗ Inflation starting at $a = 0$ in GR (quantum interference)
  - ✗ Eternal inflation
  - ✗ Exactly cyclic universes
  - ✗ **Lorentzian approaches to the big bang in infinitely higher Riemann theory**

Thank you for your attention!

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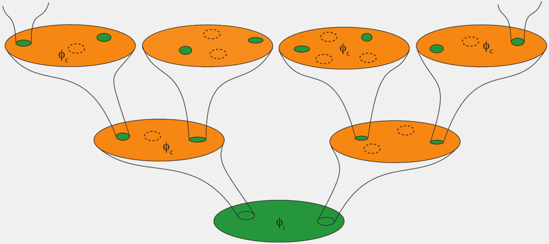
Additional slides

The story is not that simple for inflation

- If inflation really goes all the way back to the big bang singularity ($a = 0$), instabilities in the perturbations arise (quantum interference among different saddle points) $\Rightarrow$ unviable Di Tucci, Feldbrugge, Lehners & Turok [1906.09007]
- If inflation is eternal (potential is so flat that field stochastically jumps up the potential and keeps inflating), action is divergent



$$S_{\text{on-shell}} \sim \int_0^\infty \mathrm{d}t\, a^3\, V(\phi)\, \underbrace{\mathrm{Prob}[\phi \text{ is inflating at time } t]}_{\text{sol. to Fokker-Planck equation}} \to \infty \quad \text{if} \quad \frac{|V_{,\phi}|}{V^{3/2}} < \frac{1}{\sqrt{2}\pi}$$

$\rightarrow$ Reminiscent of swampland criteria Rudelius [1905.05198]