Challenges for the Matter Bounce Scenario

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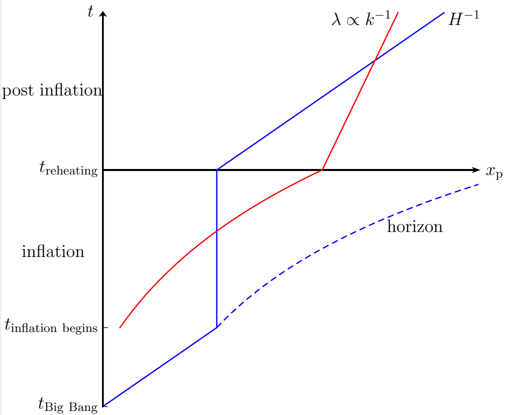
Based on work with **Robert Brandenberger** (McGill U.), **Yi-Fu Cai** (USTC, Hefei, China), and other collaborators

PRD **92**, 063532 (2015) [arXiv:1508.04141]
JCAP **1611**, 029 (2016) [arXiv:1609.02556]
JCAP **1703**, 031 (2017) [arXiv:1612.02036]
JCAP **1801**, 011 (2018) [arXiv:1711.10472]

Introduction and Motivation

- We search for a causal mechanism that can explain the observed large scale structures of our universe from primordial fluctuations
- The standard picture is 'horizon exit' and 'horizon re-entry'
- E.g., inflation: $a(t) \propto e^{Ht}$, $H \simeq \mathrm{const.}$, $H^{-1} = \mathrm{Hubble\ radius}$



Successes of Inflation

Inflation explains:

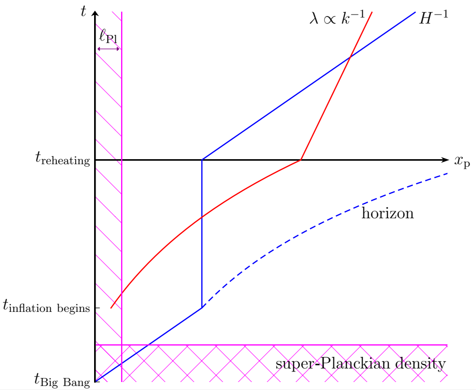
- the formation of structure problem
- the horizon problem
- the flatness problem
- the monopole problem

Also, it gives (in general):

- nearly scale-invariant power spectra of curvature and tensor perturbations
- small non-Gaussianities

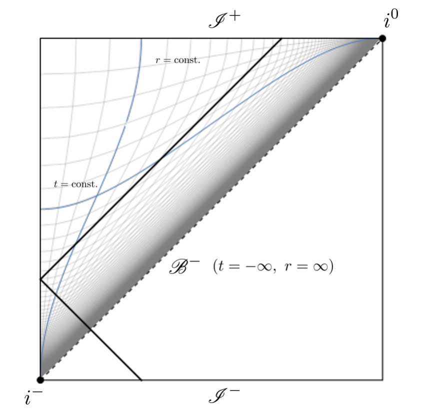
Problems with Inflation

- Trans-Planckian problem Brandenberger & Martin [hep-th/0005209, hep-th/0410223]
- Singularity problem Borde & Vilenkin [gr-qc/9612036]; Borde *et al.* [gr-qc/0110012]; Yoshida & JQ [1803.07085]
- Unpredictability, unlikeliness, initial conditions problem, measure problem Ijjas, Steinhardt, & Loeb [1304.2785, 1402.6980]
- and more conceptual issues Brandenberger [1203.6698]



Aside: Inextendibility of Inflation

- de Sitter spacetime ($a(t) = a_0 e^{Ht}$ in FRW) has a past boundary at finite affine length (when $a \to 0$)
- Yet, the full spacetime is geodesically complete. It can be extended in the appropriate coordinate basis (global coordinates, not FRW)

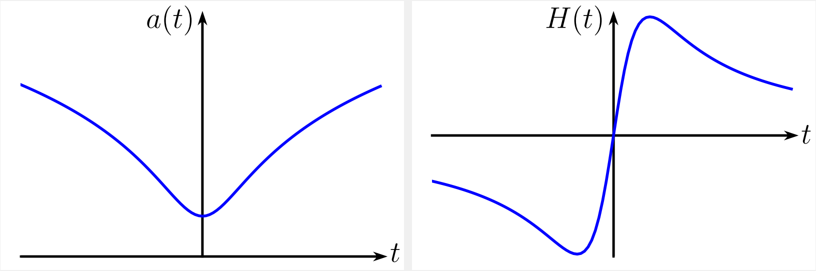


Aside: Inextendibility of Inflation

- Similarly, any spacetime is extendible iff $\boldsymbol{Ricci}$ is finite in the appropriate basis (not FRW!)
- Result: extendible iff $\dot{H}/a^2$ is finite Yoshida & JQ [1803.07085]
- Consequence: if leading energy component is quasi-de Sitter ($p \simeq -\rho$), then the spacetime is extendible only if sub-leading component severely violates the Null Energy Condition ($p \leq -(5/3)\rho$)
- E.g. 1: Starobinsky inflation is inextendible $\rightarrow$ paralarallely propagated curvature singularity
- E.g. 2: Small field inflation is C^0 extendible, but unstable against initial condition fluctuations Brandenberger [1601.01918]

Alternatives to Inflation

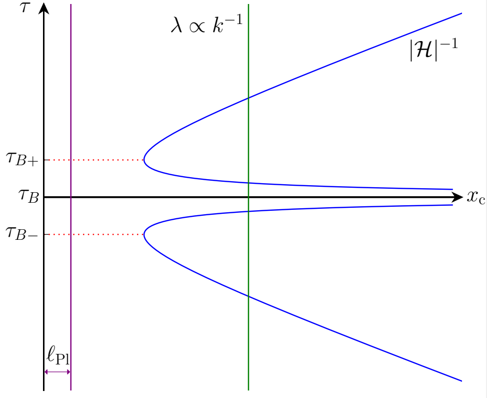
- There are a few alternative scenarios for the very early universe
- E.g., **nonsingular bouncing cosmology** (Ekpyrotic scenario, matter bounce scenario, pre-Big Bang scenario)



$\rightarrow$ Need theory beyond Einstein gravity to avoid singularity

Nonsingular Bouncing Cosmology

- Solves the problems of standard Big Bang cosmology
- Free of the trans-Planckian problem
- Can avoid the initial Big Bang singularity



What about the connection to observations?

Matter Bounce Scenario

- Perturbations exit the Hubble radius in a **matter-dominated contracting phase**, when $a(\tau) \propto \tau^2$ $(p = 0)$
- With an initial quantum vacuum, curvature perturbations have a scale-invariant primordial power spectrum Wands [gr-qc/9809062]; Finelli & Brandenberger [hep-th/0112249]

$$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right) v_k = 0\,, \qquad {}' \equiv \frac{d}{d\tau}$$

$$v_k = a\mathcal{R}_k\sqrt{2\epsilon} = \sqrt{3}a\mathcal{R}_k\,, \qquad \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}\left(1 + \frac{p}{\rho}\right) = \frac{3}{2}$$

- Same for tensor modes:

$$u_k'' + \left(k^2 - \frac{2}{\tau^2}\right) u_k = 0\,, \quad u_k = \frac{1}{2}ah_k$$

- Advantage: implementable with a single canonical scalar field; only adiabatic perturbations

Power Spectra for the Matter Bounce

- Power spectra:

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 = \frac{1}{48\pi^2}\frac{H_{B-}^2}{M_{\mathrm{Pl}}^2}$$

$$\mathcal{P}_{\mathrm{t}}(k) \equiv 2 \times \frac{k^3}{2\pi^2}|h_k|^2 = \frac{1}{2\pi^2}\frac{H_{B-}^2}{M_{\mathrm{Pl}}^2}$$

- Tensor-to-scalar ratio:

$$r \equiv \frac{\mathcal{P}_{\mathrm{t}}}{\mathcal{P}_{\mathcal{R}}} = 24$$

- Observations: $r < 0.07\ (2\sigma)$ BICEP2 [1510.09217]

$\longrightarrow$ **Ruled out!**

Possible Resolution #1

- What if $c_\mathrm{s} \ll 1$, e.g. with a k-essence scalar field?
- Curvature perturbations are amplified:

$$v_k'' + \left(c_\mathrm{s}^2 k^2 - \frac{2}{\tau^2}\right) v_k = 0 \implies \mathcal{P}_\mathcal{R}(k) = \frac{1}{48\pi^2 c_\mathrm{s}} \frac{H_{B-}^2}{M_\mathrm{Pl}^2} \implies r = 24 c_\mathrm{s}$$

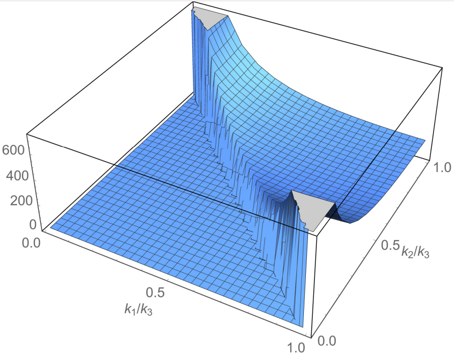
- $r < 0.07 \iff c_\mathrm{s} \lesssim 0.003$
- But $c_\mathrm{s} \ll 1 \implies$ strong coupling Baumann *et al.* [1101.3320]
- So the scalar three-point function is also amplified Li, JQ *et al.* [1612.02036]

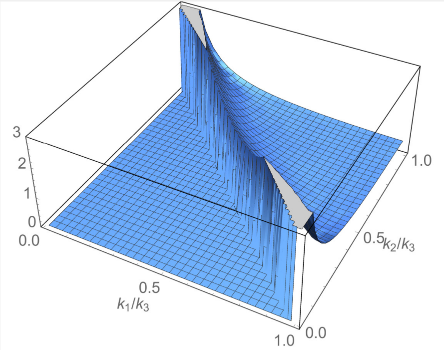
$$f_\mathrm{NL}^\mathrm{local} \simeq -\frac{165}{16} + \frac{65}{8 c_\mathrm{s}^2}$$

- Cannot simultaneously satisfy observational bound on r and $f_\mathrm{NL}^\mathrm{local} = 0.8 \pm 5.0\ (1\sigma)$ Planck [1502.01592]
- Also, $c_\mathrm{s} \ll 1$ with a fluid $\implies$ Jeans (gravitational) instability $\implies$ black hole formation (see ITC Luncheon talk!) JQ & Brandenberger [1609.02556]

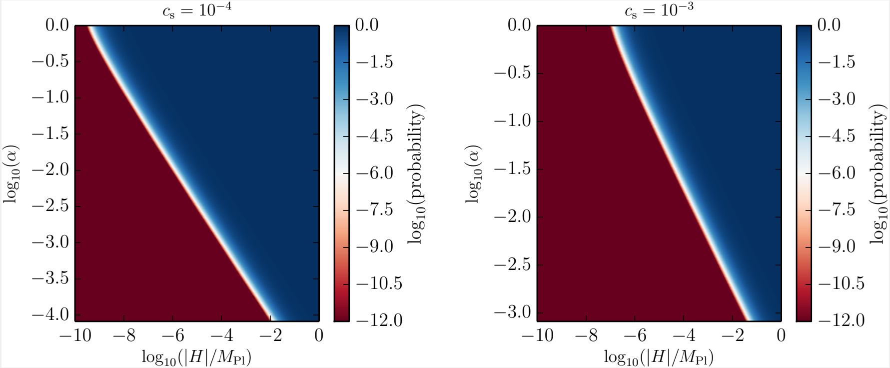
Small sound speed: non-Gaussianities

- Non-Gaussianity dimensionless shape function for $c_s = 0.2$ (left plot) and $c_s = 0.87$ (right plot)





Small sound speed: black hole formation



$$\alpha \equiv R_{\rm BH}/|H|^{-1}$$

Possible Resolution #2

- What if $\mathcal{R}$ grows during the nonsingular bounce phase?

$$\frac{r_{\text{before bounce}}}{r_{\text{obs}}} = \left| 1 + \frac{\Delta\mathcal{R}}{\mathcal{R}_{\text{before bounce}}} \right|^2$$

- Creates large non-Gaussianities JQ *et al.* [1508.04141]

$$f_{\text{NL}} \propto \left(\frac{\Delta\mathcal{R}}{\mathcal{R}_{\text{before bounce}}} \right)^{\#}$$

- $\implies$ cannot simultaneously satisfy observational constraints on r and f_{NL}

Matter Bounce No-Go Theorem

- A lower bound on the amplification of curvature perturbations $\mathcal{R}$
  $\Longleftrightarrow$ an upper bound on the tensor-to-scalar ratio r
  $\Longleftrightarrow$ a lower bound on primordial non-Gaussianities f_{NL}

- With Einstein gravity + a single (not necessarily canonical) scalar field:

satisfying the current observational upper bound on r cannot be done without contradicting the current observational constraints on f_{NL} (and vice versa) JQ *et al.* [1508.04141]; Li, JQ *et al.* [1612.02036]

Evading the No-Go Theorem

- Multiple fields, e.g., matter bounce curvaton scenario (entropy modes sourcing curvature perturbations) Cai *et al.* [1101.0822]
- Or with a single field, go beyond Einstein gravity
  $\longrightarrow$ need to modify tensor modes
- Add a nontrivial mass m_g to the graviton:

$$\mathcal{L}^{(2)}_{\mathrm{tensor}} \supset a^2 \left[\left(h'_{ij} \right)^2 - \left(\nabla h_{ij} \right)^2 \right] \to a^2 \left[\left(h'_{ij} \right)^2 - \left(\nabla h_{ij} \right)^2 - m_g^2 a^2 h_{ij}^2 \right]$$

$$\Longrightarrow \; u''_k + \left(k^2 + m_g^2 a^2 - \frac{a''}{a} \right) u_k = 0$$

Tensor Power Spectrum with Massive Gravity

- $m_g \ll |H_*|$ at horizon exit:

$$\mathcal{P}_\mathrm{t}(k) \simeq \frac{1}{2\pi^2}\left(\frac{k}{aM_\mathrm{Pl}}\right)^{n_\mathrm{t}}\left|\frac{H_{B-}}{M_\mathrm{Pl}}\right|^{2-n_\mathrm{t}}, \qquad n_\mathrm{t} \simeq \frac{8m_g^2}{3H_*^2} \ll 1$$

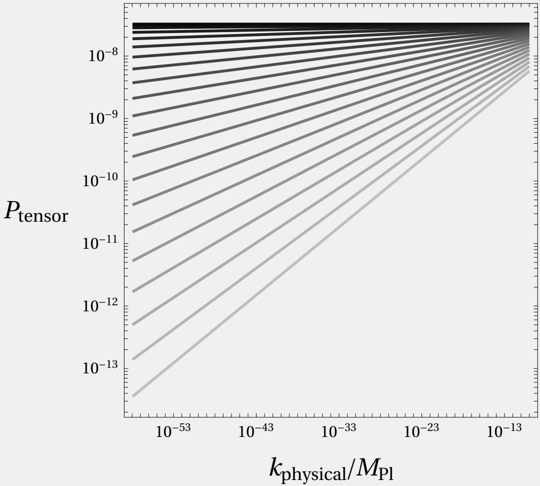
- $m_g \gg |H_*|$ at horizon exit:

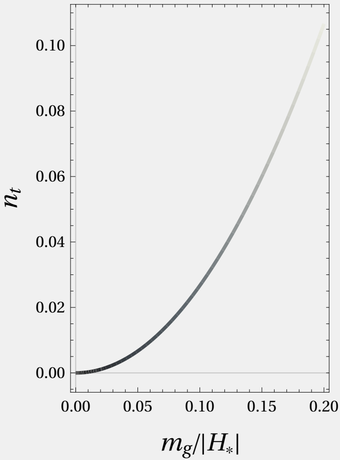
$$\mathcal{P}_\mathrm{t}(k) \simeq \frac{2}{\pi^2}\left(\frac{k}{a}\right)^3 \frac{1}{M_\mathrm{Pl}^2 m_g}$$

- In both cases, $r(0.05\,\mathrm{Mpc}^{-1}) \ll 0.07$ Lin, JQ & Brandenberger [1711.10472]

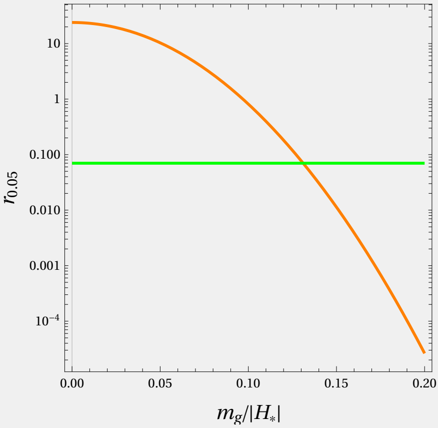
Tensor Power Spectrum with Massive Gravity

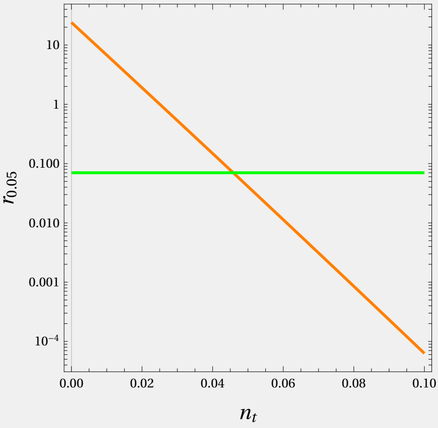
- Note that $0.05\,\mathrm{Mpc}^{-1} \approx 10^{-58}\,M_{\mathrm{Pl}}$





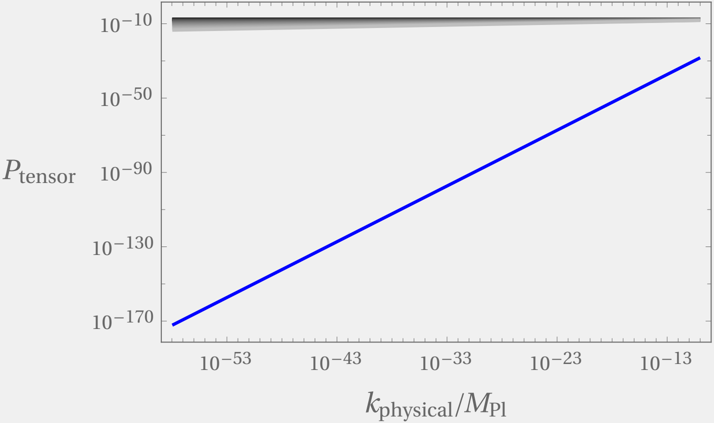
Tensor Power Spectrum with Massive Gravity





Tensor Power Spectrum with Massive Gravity

- If $m_g \gg |H(t)|$, $r \approx 0$



Anisotropy problem

- Consider

$$ds^2 = -\mathrm{d}t^2 + a^2 \sum_{i=1}^{3} e^{2\theta_i} (\mathrm{d}x^i)^2 \,, \qquad \sum_{i=1}^{3} \theta_i = 0$$

- Einstein gravity = Friedmann equations + anisotropies:

$$\ddot{\theta}_i + 3H\dot{\theta}_i = 0 \implies \dot{\theta}_i \sim a^{-3} \implies \rho_\theta \sim \sum_{i=1}^{3} \dot{\theta}_i^2 \sim a^{-6}$$

- Analogous to a massless scalar field

$$\mathcal{L}_\theta = -\frac{1}{2}\partial_\mu \theta \partial^\mu \theta \implies p_\theta = \rho_\theta \; (w = 1)$$

- Anisotropies dominate at high energies:

$$H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\left(\frac{\rho_{\mathrm{m}}^0}{a^3} + \frac{\rho_{\mathrm{rad}}^0}{a^4}\right) - \frac{k}{a^2} + \frac{\Lambda}{3} + \frac{\rho_\theta^0}{a^6}$$

- Not a problem for Ekpyrosis which has $w \gg 1$ Garfinkle *et al.* [0808.0542]

Anisotropies with Massive Gravity

- With a massive graviton Lin, JQ & Brandenberger [1711.10472]

$$\mathcal{L}_\theta = -\frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \frac{1}{2}m_g^2\theta^2$$

$$\longrightarrow \ddot{\theta}_i + 3H\dot{\theta}_i + m_g^2\theta_i = 0\,, \qquad \rho_\theta \sim \sum_{i=1}^{3}(\dot{\theta}_i^2 + m_g^2\theta_i^2)$$

- $m_g \gg |H| \implies \rho_\theta \sim a^{-3}$ (i.e., $p_\theta = 0$)

- No anisotropy problem anymore:

$$H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\left(\frac{\rho_{\mathrm{m}}^0}{a^3} + \frac{\rho_{\mathrm{rad}}^0}{a^4}\right) - \frac{k}{a^2} + \frac{\Lambda}{3} + \frac{\rho_\theta^0}{a^3}$$

- Would need $m_g \gg |H_{B-}|$, but $m_g < 7.2\times 10^{-23}\,\mathrm{eV}\ (2\sigma)$ today $\longrightarrow$ requires symmetry breaking or $m_g(t)$ $\longrightarrow$ might be natural with chameleon coupling and solve other problems of massive gravity like the Higuchi bound De Felice *et al.* [1711.04655]

Effective Massive Gravity Action

- ADM decomposition:

$$ds^2 = -N^2 \mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)$$

- Effective massive gravity action:

$$S = \int \mathrm{d}t\, \mathrm{d}^3\boldsymbol{x}\, \sqrt{\gamma} \left(N \left(\frac{M_{\mathrm{Pl}}^2}{2} R - \Lambda \right) - m_g^2 V[\gamma_{ij}] \right)$$

- The nonderivative potential $V[\gamma_{ij}]$ is independent of the lapse N

  $\longrightarrow$ **only 2 DoF propagate (2 polarization states of GWs), but diffeomorphism invariance is broken** Comelli *et al.* [1407.4991]

Effective Massive Gravity Action

- In the EFT language, take

$$V[\gamma_{ij}] \sim M_{\mathrm{Pl}}^2 \bar{\delta}\gamma^{ij}\bar{\delta}\gamma_{ij}$$

  where $\bar{\delta}\gamma_{ij}$ denotes the traceless part of the linear perturbations of the spatial metric Lin & Labun [1501.07160]

- $\Longrightarrow$ background unaffected (Friedmann equations unchanged)
  scalar and vector perturbations unaffected
  tensor perturbations and anisotropies receive a mass term (m_g)

- Can be implemented with Stückelberg scalar fields (additional slides)

Conclusions

- The matter bounce scenario is an alternative to inflation
- It naturally predicts a very large tensor-to-scalar ratio (r)
- r cannot be suppressed by enhancing curvature perturbations; otherwise f_{NL} is too large
- Tensor modes can be suppressed with a (Lorentz-violating) massive graviton
- Anisotropies are suppressed likewise, but need $m_g(t)$
- In sum, the simple idea of the matter bounce doesn't fit with observations; needs nontrivial theory to work
- Motivates us to keep looking for and testing alternative scenarios for the very early universe

Acknowledgments

Thank you for your attention!

I acknowledge support from the following agencies:

Additional Slides: Action with Stückelberg Scalar Fields

- Introduce 1 timelike (ϕ^0) and 3 spacelike (ϕ^i) Stückelberg scalar fields with the following VEVs:

$$\phi^0 = f(t)\,, \qquad \phi^i = x^i$$

- Impose symmetries:

$$\phi^i \to \Lambda^i{}_j \phi^j\,, \qquad \phi^i \to \phi^i + \Xi^i(\phi^0)$$

$$\Lambda^i{}_j :\ SO(3)\ \text{rotation operator;}\ \Xi^i :\ \text{generic function of}\ \phi^0$$

Additional Slides: Action with Stückelberg Scalar Fields

- The following quantities are invariant under the symmetries Dubovsky *et al.* [hep-th/0411158]

$$X = g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^0$$

$$Z^{ij} = g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j - \frac{(g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^i)(g^{\alpha\beta}\partial_\alpha\phi^0\partial_\beta\phi^j)}{X}$$

- The following operator is traceless Lin & Sasaki [1504.01373]

$$\bar{\delta}Z^{ij} = \frac{Z^{ij}}{Z} - 3\frac{\delta_{k\ell}Z^{ik}Z^{j\ell}}{Z^2}\,, \qquad Z = \delta_{ij}Z^{ij}$$

- Construct quadratic operator graviton mass term:

$$\mathcal{L}_{\mathrm{mass}} \sim M_{\mathrm{Pl}}^2 m_g^2 \delta_{ik}\delta_{j\ell}(\bar{\delta}Z^{ij})(\bar{\delta}Z^{k\ell})$$

- Resulting theory has 2 gravitational DoF Lin, JQ & Brandenberger [1711.10472]