On the stability of nonsingular cosmologies: the case of limiting curvature

Based on work with Robert Brandenberger (McGill), Masahide Yamaguchi (Tokyo Tech.), and Daisuke Yoshida (McGill)

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Outline

- Introduction and motivation
- Nonsingular cosmology with a Galileon scalar field
- Nonsingular cosmology with limiting curvature
- Conclusions

Motivation

- General Relativity (GR) with normal matter
  $\Longrightarrow$ cosmological singularities are inevitable
  (Penrose ['65], Hawking ['67])

- Even inflationary cosmology (within GR) is inevitably past incomplete
  (Borde & Vilenkin [gr-qc/9312022], Border et al. [gr-qc/0110012])

- One would thus like to build a theory that is free of these bad singularities
  $\Longrightarrow$ one has to go beyond classical GR

- Ultimate goal: a theory of the very early universe embedded in a quantum theory of gravity without singularities

Alternatives to inflation are often nonsingular

- Emerging scenarios:
  - String Gas Cosmology
    (Brandenberger & Vafa ['89], Brandenberger [1505.02381])
  - Galilean Genesis
    (Creminelli et al. [1007.0027], Nishi & Kobayashi [1501.02553])

- Bouncing scenarios:
  - Pre-Big Bang cosmology
    (Gasperini & Veneziano [hep-th/9211021, hep-th/0207130])
  - Ekpyrotic scenario
    (Khoury et al. [hep-th/0103239], Lehners [0806.1245])
  - Matter Bounce Cosmology
    (Wands [gr-qc/9809062], Finelli & Brandenberger [hep-th/0112249], Brandenberger [1206.4196])

Approaches to nonsingular cosmology

- Quantum Gravity:
  - Loop Quantum Cosmology (Wilson-Ewing [1211.6269], Cai & Wilson-Ewing [1402.3009])
  - String Theory (Cheung et al. [1601.03807])
  - Group Field Theory (Oriti et al. [1602.08271], Sakellariadou [1703.09498])
- Matter violating the Null Energy Condition (NEC):
  - Quintom matter (Cai et al. [0704.1090])
  - Lee-Wick theory (Cai et al. [0810.4677])
  - Ghost Condensate (Lin et al. [1007.2654])
  - Galileon scalar field (Qiu et al. [1108.0593], Easson et al. [1109.1047], Cai et al. [1206.2382], Battarra et al. [1404.5067])
- Modified Gravity:
  - $f(R)$ gravity (Bamba et al. [1309.3748])
  - $f(T)$ gravity (Cai et al. [1104.4349], Amoros et al. [1305.2344])
  - Gauss-Bonnet gravity (Bamba et al. [1403.3242, 1411.3852])
  - Hořava-Lifshitz gravity (Brandenberger [0904.2835])
- Effective Field Theory (EFT) (Cai et al. [1610.03400, 1701.04330], Creminelli et al. [1610.04207])

$\longrightarrow$ **Different approaches can lead to different predictions**

Example: Galileon scalar field

- Cubic Galileon:

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2} R + K(\phi, X) - G(\phi, X)\Box\phi\,,$$

$$X \equiv -\frac{1}{2} g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\,, \qquad \Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$$

- Example:

$$K(\phi, X) = [1 - g(\phi)]X + \beta X^2 - V(\phi)\,, \qquad G(\phi, X) = \gamma X$$

- $g(\phi) > 1 \implies$ ghost condensate $\implies$ NEC violation
  $\implies$ nonsingular bounce

Cosmological perturbations

- Perturbed metric and field:

$$g_{\mu\nu}(t,\mathbf{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\mathbf{x}) \,, \qquad \phi(t,\mathbf{x}) = \bar{\phi}(t) + \delta\phi(t,\mathbf{x})$$

- Unitary gauge: $\delta\phi(t,\mathbf{x}) = 0$ and

$$\delta g_{ij} = a^2(1-2\zeta)\delta_{ij} + a^2 h_{ij} \,,$$

  where $\nabla_i h^{ij} = 0$ and $h_i{}^i = 0$.
- $\zeta(t,\mathbf{x})$: curvature perturbation; $h_{ij}(t,\mathbf{x})$: tensor perturbation
- 2nd-order perturbed actions:

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}^4x\, a^3 \left[\dot{h}_{ij}^2 - \frac{1}{a^2}(\vec{\nabla} h_{ij})^2\right] \,,$$

$$S_S^{(2)} = \int \mathrm{d}^4x\, a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\vec{\nabla}\zeta)^2\right]$$

Equations of motion and (in)stability

$$S_S^{(2)} = \int \mathrm{d}^4x\, a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\vec{\nabla}\zeta)^2\right]$$

- Conditions:

$$\mathcal{G}_S > 0 \Leftrightarrow \text{no ghost instability}\,, \quad \mathcal{F}_S > 0 \Leftrightarrow \text{no gradient instability}$$

- Scalar equation of motion in Fourier space:

$$\zeta_k'' + \frac{\left(z^2\right)'}{z^2}\zeta_k' + c_\mathrm{s}^2 k^2 \zeta_k = 0\,,$$

where $c_\mathrm{s}^2 \equiv \mathcal{F}_S/\mathcal{G}_S$, $z \equiv \sqrt{2}a(\mathcal{F}_S\mathcal{G}_S)^{1/4}$, $\mathrm{d}\tau \equiv a^{-1}\mathrm{d}t$, $' \equiv \mathrm{d}/\mathrm{d}\tau$.

- If $\mathcal{F}_S < 0$, then $c_\mathrm{s}^2 < 0$, and for $|c_\mathrm{s}|k \to \infty$,

$$\zeta_k'' - |c_\mathrm{s}|^2 k^2 \zeta_k \simeq 0$$

$$\Longrightarrow \; \zeta_k(t) \sim \exp\left(k\int \mathrm{d}t\, \frac{|c_\mathrm{s}(t)|}{a(t)}\right) \longrightarrow \; \text{instability}$$

No-Go Theorems

- Within Horndeski theories ($\Longleftrightarrow$ generalized Galileon $\supset$ cubic Galileon), it is not possible to have a geodesically complete spacetime and be free of both ghost and gradient instabilities at all times ($\mathcal{G}_S, \mathcal{F}_S, \mathcal{G}_T, \mathcal{F}_T > 0$). (Libanov et al. [1605.05992], Kobayashi [1606.05831])

- Horndeski theories:

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

$$+G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]\,,$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$.

- Also within EFT (Cai et al. [1610.03400, 1701.04330], Creminelli et al. [1610.04207])

$$S = \int \mathrm{d}^4x\, N\sqrt{h}\Big[\frac{1}{2}M_{\rm Pl}^2\Big({}^{(3)}R + \frac{E_{ij}E^{ij} - E^2}{N^2}\Big) - \frac{M_{\rm Pl}^2\dot{H}}{N^2} - M_{\rm Pl}^2(3H^2 + \dot{H})$$

$$+\frac{1}{2}m_2^4\delta N^2 - \hat{m}_1^3\delta N\delta E - \frac{1}{2}\bar{m}_1^2\delta E^2 - \frac{1}{2}\bar{m}_2^2\delta E^i{}_j\delta E^j{}_i + ...\Big]$$

Evading the No-Go Theorems

- In EFT, include the operator ${}^{(3)}R\delta N$.

- Work with beyond-Horndeski theories
(Gleyzes et al. [1404.6495, 1408.1952], Gao [1406.0822, 1409.6708], Kobayashi et al. [1504.05710])

- Ijjas & Steinhardt [1609.01253] found a way out with only a quartic Galileon, but their action vanishes at one instant in time, i.e.

$$\mathcal{G}_T(t_\gamma) = \mathcal{F}_T(t_\gamma) = \mathcal{G}_S(t_\gamma) = \mathcal{F}_S(t_\gamma) = 0\,.$$

Gauge issue or physical pathology?

Other approach to nonsingular cosmology: limiting curvature

- The idea of limiting curvature: there should exist a fundamental length scale ℓ_f (possibly $\sim \ell_{\rm Pl}$) such that

$$|R| < \ell_f^{-2},\ |R_{\mu\nu}R^{\mu\nu}| < \ell_f^{-4},\ |\nabla_\rho R_{\mu\nu}\nabla^\rho R^{\mu\nu}| < \ell_f^{-6},$$

$$|C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}| < \ell_f^{-8},\ \text{etc}\,.$$

- Difficulty: one could have $|R_{\mu\nu}R^{\mu\nu}| < \ell_f^{-4}$, but still $|\nabla_\rho R_{\mu\nu}\nabla^\rho R^{\mu\nu}| \to \infty$.
- Limiting curvature hypothesis: find a theory with a finite number of curvature invariants bounded, e.g., $|R| \leq \ell_f^{-2}$, $|R_{\mu\nu}R^{\mu\nu}| \leq \ell_f^{-4}$. Then, when these invariants take on their limiting values, any solution of the field equations reduces to a definite nonsingular solution.

Mukhanov & Brandenberger [PRL, 1992]

Limiting curvature implementation: example in special relativity

- Action for a point particle in special relativity:

$$S = m \int \mathrm{d}t \left[\frac{1}{2}\dot{x}^2 + \varphi \dot{x}^2 - V(\varphi) \right], \qquad V(\varphi) = \frac{2\varphi^2}{1+2\varphi}.$$

- $\delta_\varphi S = 0 \implies \dot{x}^2 = \frac{\mathrm{d}V}{\mathrm{d}\varphi} = 1 - \frac{1}{(1+2\varphi)^2} \implies \dot{x}^2 \leq 1 \; \forall \varphi \in (-\infty, \infty)$
- Solving for φ in terms of $\dot{x}^2$ and substituting in the action above, one finds

$$S = m \int \mathrm{d}t \, \sqrt{1 - \dot{x}^2}\,,$$

as expected.

Limiting curvature implementation

- Naturally constructed to avoid singularities (contrary to, e.g., Galileons)
- Used to construct nonsingular black holes (Frolov et al. ['89, '90], Morgan ['91], Trodden et al. ['93], Bogojevic & Stojkovic ['00], Easson ['03], Frolov ['16], Chamseddine & Mukhanov ['17])
- In cosmology, the action is (Mukhanov & Brandenberger ['92], Brandenberger et al. [gr-qc/9303001])

$$S = \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,[R + \chi_1 I_1 - V_1(\chi_1) + \chi_2 I_2 - V_2(\chi_2)] + S_{\mathrm{m}}\,,$$

  where $I_1,\ I_2$ can be functions of R, $R_{\mu\nu}R^{\mu\nu}$, $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$, etc.
- Assume $I_1,\ I_2 \sim \mathcal{O}(R)$. Since $\bar{R} = 12H^2 + 6\dot{H}$, a natural choice is

$$\bar{I}_1 = 12H^2\,, \qquad \bar{I}_2 = -6\dot{H}\,.$$

- So $\delta_{\chi_1}S = 0$ and $\delta_{\chi_2}S = 0$ gives the constraint equations

$$\bar{I}_1 = 12H^2 = \frac{\mathrm{d}V_1}{\mathrm{d}\chi_1}\,, \qquad \bar{I}_2 = -6\dot{H} = \frac{\mathrm{d}V_2}{\mathrm{d}\chi_2}\,.$$

Limiting curvature implementation: cosmology

- Recover Einstein gravity (Minkowski in vacuum, FRW with matter) $\longrightarrow$ as $|\chi_n| \ll 1$, require $V_n(\chi_n) \sim \chi_n^2$

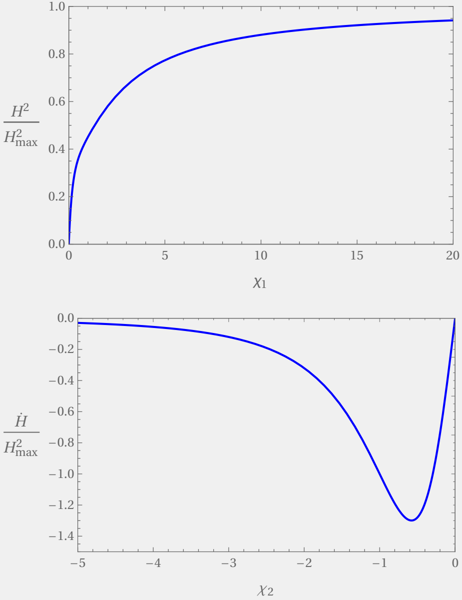
- $V_1' \to \mathrm{const}$ as $|\chi_1| \to \infty$, $V_2' \to 0$ as $|\chi_2| \to \infty$
  $\implies |H| \leq \max$ and $\dot{H} \to 0$
  $\implies$ asymptotically de Sitter

- E.g.,

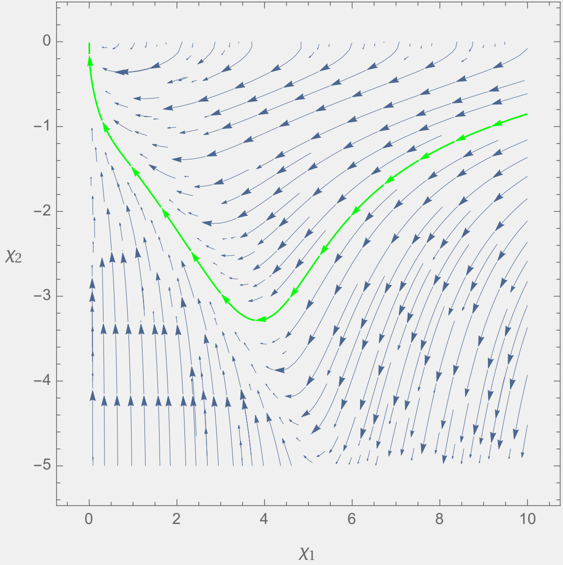
$$V_1(\chi_1) = 12H_{\max}^2 \frac{\chi_1^2}{1+\chi_1}\left(1 - \frac{\ln(1+\chi_1)}{1+\chi_1}\right),$$

$$V_2(\chi_2) = -12H_{\max}^2 \frac{\chi_2^2}{1+\chi_2^2}\,.$$

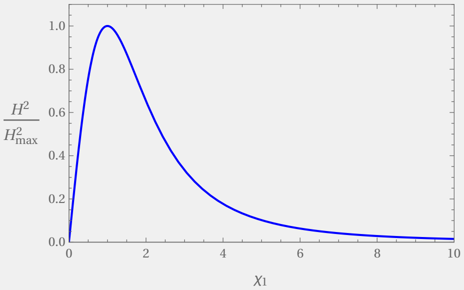
Example: de Sitter to Minkowski (inflation)

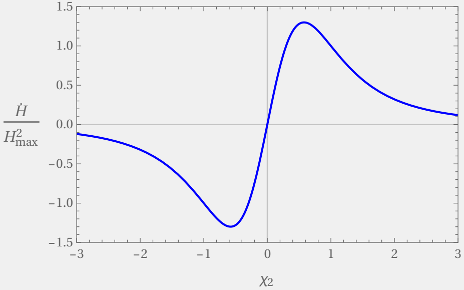


Example: de Sitter to Minkowski (inflation)

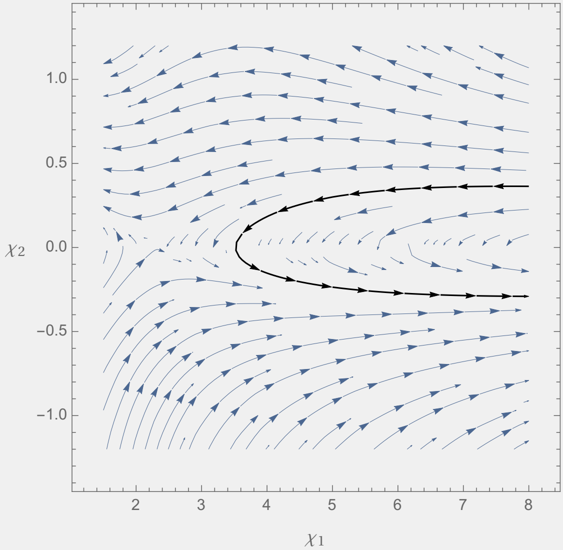


Example: Minkowski to Minkowski (genesis)





Example: Minkowski to Minkowski (genesis)



Perturbations and stability

$$\mathcal{L} = R + \chi_1 I_1 - V_1(\chi_1) + \chi_2 I_2 - V_2(\chi_2)$$

- Mukhanov & Brandenberger ['92] took

$$I_2 \equiv \sqrt{12 R_{\mu\nu} R^{\mu\nu} - 3R^2}\,, \qquad I_1 \equiv I_2 + R\,.$$

- One can check that $12 R_{\mu\nu} R^{\mu\nu} - 3R^2 \geq 0$ for any spherically symmetric metric
- $I_2 = 0$ only for Minkowski and de Sitter
- In an FLRW background, $\bar{I}_1 = 12H^2$ and $\bar{I}_2 = -6\dot{H}$ as wanted

Perturbations and stability

$$\mathcal{L} = R + \chi_1 \left(R + \sqrt{12 R_{\mu\nu} R^{\mu\nu} - 3R^2} \right) - V_1(\chi_1)$$
$$+ \chi_2 \sqrt{12 R_{\mu\nu} R^{\mu\nu} - 3R^2} - V_2(\chi_2)$$

- Consider tensor perturbations:

$$\delta g_{ij} = a^2 h_{ij}$$

- 2nd-order perturbed action (in Fourier space):

$$S_T^{(2)} \supset \int \mathrm{d}t \mathrm{d}^3 k \, a^3 \left(\mathcal{G}_T \ddot{h}_k^2 + \mathcal{K}_T \dot{h}_k^2 - \mathcal{M}_T \frac{k^2}{a^2} h_k^2 \right) ,$$

$$\mathcal{G}_T = -\frac{\chi_1 + \chi_2}{2\dot{H}} .$$

$\longrightarrow$ Ostrogradski instability (corresponds to a linearly unstable Hamiltonian)

Possible resolution

$$\mathcal{L} = R + \chi_1 I_1 - V_1(\chi_1) + \chi_2 I_2 - V_2(\chi_2)$$

- Include the Weyl tensor squared:

$$I_2 \equiv \sqrt{12 R_{\mu\nu}R^{\mu\nu} - 3R^2 + 3\kappa C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}}\,, \qquad I_1 \equiv I_2 + R\,.$$

- Perturbing the action in the tensor sector:

$$S_T^{(2)} \supset \int \mathrm{d}t\mathrm{d}^3k\, a^3 \left(\mathcal{G}_T \ddot{h}_k^2 + \mathcal{K}_T \dot{h}_k^2 - \mathcal{M}_T \frac{k^2}{a^2} h_k^2\right)\,,$$

$$\mathcal{G}_T = -(2+\kappa)\frac{\chi_1 + \chi_2}{4\dot{H}}\,,$$

$\longrightarrow \kappa = -2 \implies \mathcal{G}_T = 0 \longrightarrow$ no Ostrogradski ghost

- Is it valid at higher order ($S_T^{(3)}$, $S_T^{(4)}$, ...)?

Perturbations modes

$$\mathcal{L} = R + \chi_1 \left(R + \sqrt{12 R_{\mu\nu} R^{\mu\nu} - 3R^2 - 6C^2} \right) - V_1(\chi_1)$$
$$+ \chi_2 \sqrt{12 R_{\mu\nu} R^{\mu\nu} - 3R^2 - 6C^2} - V_2(\chi_2) \, , \qquad C^2 \equiv C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$$

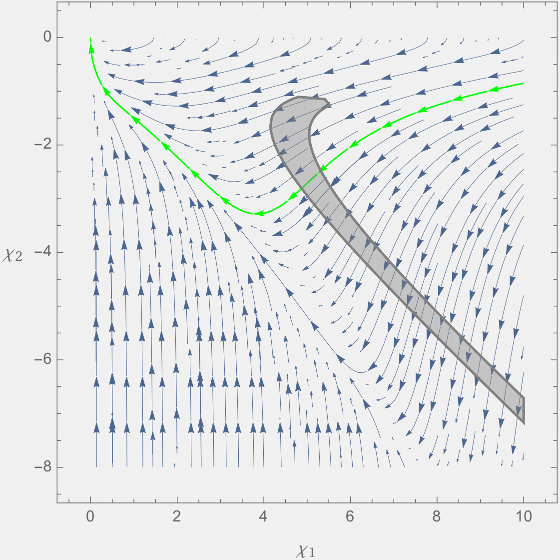
- No propagating vector modes
- Tensor and scalar modes:

$$S_T^{(2)} \sim \int \mathrm{d}t \mathrm{d}^3 k \, a^3 \left(\mathcal{K}_T \dot{h}_k^2 - \mathcal{M}_T \frac{k^2}{a^2} h_k^2 \right)$$

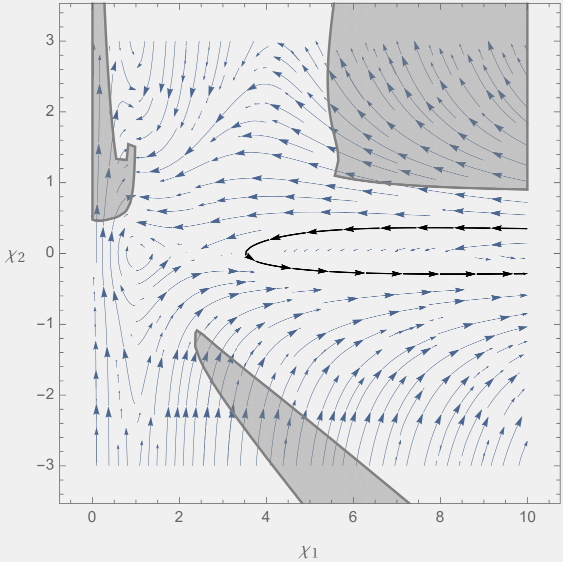
$$S_S^{(2)} \sim \int \mathrm{d}t \mathrm{d}^3 k \, a^3 \left(\mathcal{K}_S \dot{\Phi}_k^2 - \mathcal{M}_S(k) \Phi_k^2 \right)$$

- $\mathcal{M}_S(k) \sim \mathcal{O}\left(\frac{k^8}{a^8}\right)$ for $\frac{k}{a} \gg 1 \longrightarrow$ modified dispersion relation
- $\mathcal{K}_T$, $\mathcal{M}_T$, $\mathcal{K}_S$, $\mathcal{M}_S$ are complicated functions of χ_n and $V_n(\chi_n)$ that can be positive or negative depending on the background trajectory $\longrightarrow$ possible ghost and gradient instabilities

Stability during inflation



Stability during genesis



Equivalent to Gauss-Bonnet gravity

$$\mathcal{L} = R + \chi_1 \left(R + \sqrt{12 R_{\mu\nu} R^{\mu\nu} - 3R^2 - 6C^2} \right) - V_1(\chi_1)$$
$$+ \chi_2 \sqrt{12 R_{\mu\nu} R^{\mu\nu} - 3R^2 - 6C^2} - V_2(\chi_2)\,, \qquad C^2 \equiv C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$$

- The theory is simply $\mathcal{L} = f(R, \mathcal{G})$
- Indeed,

$$I_1 = R + \sqrt{R^2 - 6\mathcal{G}}\,, \qquad I_2 = \sqrt{R^2 - 6\mathcal{G}}\,,$$

  since

$$\begin{cases} \mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\ C^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2 \end{cases}$$

$$\Longrightarrow\; 6\mathcal{G} = 4R^2 - 12R_{\mu\nu}R^{\mu\nu} + 6C^2$$

- In an anisotropic background, $f(R, \mathcal{G})$ gravity has inevitable ghost degrees of freedom, which are absent on FLRW backgrounds (De Felice & Tanaka [1006.4399])

Construct another curvature invariant function

$$\mathcal{L} = R + \chi_1 I_1 - V_1(\chi_1) + \chi_2 I_2 - V_2(\chi_2)$$

- Consider first derivatives of R:

$$X^\mu{}_\nu \equiv \nabla^\mu R \nabla_\nu R\,, \qquad X = \nabla^\mu R \nabla_\mu R$$

- For a flat FLRW background,

$$\bar{X} = -36(4H\dot{H} + \ddot{H})^2$$

- Want to construct I_1 such that $\bar{I}_1 = 12H^2$:

$$I_1 \equiv -\frac{1}{X^3}[4X^2(\nabla_\mu \nabla_\nu R)^2 - 2X(\nabla_\mu X)^2 + (\nabla_\mu R \nabla_\nu X)^2]$$

- Then, $I_2 \equiv I_1 - R$ and $\bar{I}_2 = -6\dot{H}$.

Perturbations for the new curvature invariant function

- No propagating vector modes
- Tensor modes:

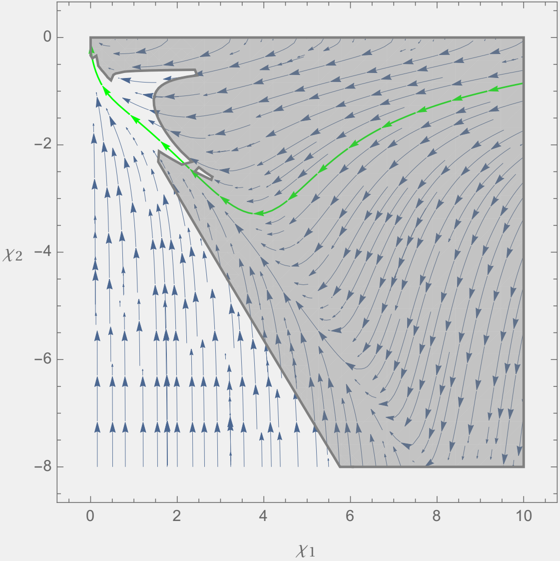
$$S_T^{(2)} \sim \int \mathrm{d}t\mathrm{d}^3k\, a^3 \left(\mathcal{K}_T \dot{h}_k^2 - \mathcal{M}_T \frac{k^2}{a^2} h_k^2 \right)$$

where

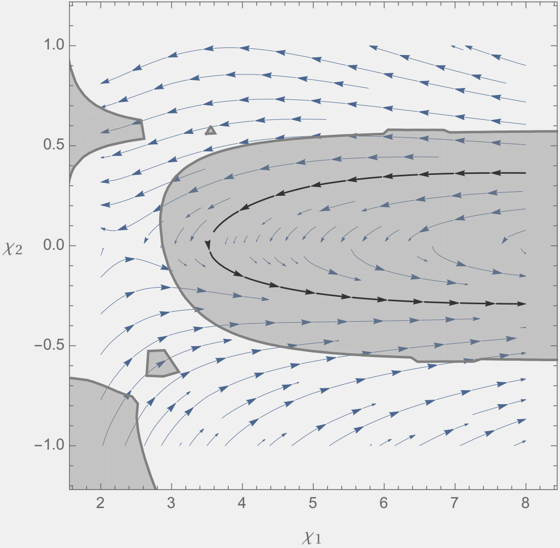
$$\mathcal{K}_T = \frac{1 + 4\chi_1 + 3\chi_2}{2}\,, \qquad \mathcal{M}_T = \frac{1 - \chi_2}{2}$$

- No Ostrogradski instability. No ghost or gradient instabilities as long as $\chi_2 < 1$ and $\chi_1 > -(1 + 3\chi_2)/2$.
- No superluminality $\implies c_\mathrm{s}^2 \equiv \mathcal{M}_T/\mathcal{K}_T \leq 1 \implies \chi_1 \geq -\chi_2$.
- Similar story in the scalar sector, though the conditions on χ_1 and χ_2 are slightly more non-trivial
  $\longrightarrow$ the theory could be used to construct stable nonsingular inflationary or genesis scenarios

Stability during inflation



Stability during genesis



Other approaches to nonsingular cosmology with limiting curvature

- Mimetic gravity (Chamseddine & Mukhanov [1612.05860])

$$S = M_{\rm Pl}^2 \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{2}R + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 1) + f(\Box\phi) \right] + S_{\rm m} \,,$$

with Born-Infeld type action,

$$f(\Box\phi) = 1 - \sqrt{1 - \frac{(\Box\phi)^2}{\rho_f}} + ...$$

- Constraint:

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = 1$$

- Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\rm Pl}^2}\rho\left(1 - \frac{\rho}{\rho_f}\right)$$

Nonsingular bouncing cosmology

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\rho\left(1 - \frac{\rho}{\rho_f}\right)$$

- As $\rho \to \rho_f$, $H \to 0$. In fact, H is maximal at $\rho = \rho_f/2$, so

$$H^2 \leq \frac{\rho_f}{12M_{\rm Pl}^2}$$

- Therefore, $|R| \lesssim \frac{\rho_f}{M_{\rm Pl}^2} \equiv \ell_f^{-2}$, $|R_{\mu\nu}R^{\mu\nu}| \lesssim \ell_f^{-4}$, and so on
  $\longrightarrow$ all curvature invariants are bounded
  $\longrightarrow$ limiting curvature hypothesis is realized
- Same modified Friedmann equation as in Loop Quantum Cosmology
  (Bodendorfer et al. [1703.10670], Liu et al. [1703.10812])
- Naturally leads to nonsingular bouncing cosmology
- Is mimetic gravity stable or unstable? (Barvinsky [1311.3111], Chaichian et al. [1404.4008], Ramazanov et al. [1601.05405], Ben Achour et al. [1602.08398], Firouzjahi et al. [1703.02923], Hirano et al. [1704.06031])

Conclusions

- Simple nonsingular cosmologies with Galileon scalar fields are unstable (no-go theorem)
  $\longrightarrow$ one needs to consider beyond-Horndeski theories
- Other approach: limiting curvature
- Old model of Mukhanov & Brandenberger has Ostrogradski instabilities
- Can be cured by including the Weyl tensor squared
  $\longrightarrow$ still important ghost and gradient instabilities
  $\longrightarrow$ equivalent to $f(R, \mathcal{G})$ gravity
- New curvature invariant constructed with derivatives of R leads to no apparant Ostrogradski instability
- Inflationary and genesis scenarios are mostly stable with regards to ghost and gradient instabilities
- Still very hard to construct stable nonsingular cosmologies

What's next?

- Construct viable nonsingular inflationary and genesis scenarios:
  - get 60 e-folds (inflation)
  - have slow-roll (inflation)
  - reach the right energy scale (genesis)
  - recover Einstein gravity sufficiently fast
  - have a reheating mechanism $\longrightarrow$ include matter after the early phase $\longrightarrow$ how does matter affect stability?

- Explore cosmological observables

- Explore nonsingular bouncing cosmology

- How does the theory with limiting curvature fit in the greater picture of scalar-tensor theories of gravity?
  $\longrightarrow$ included in beyond-Horndeski theories? in Degenerate Higher-Order Scalar-Tensor (DHOST) theories?

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