Bouncing Cosmology: the current state and the road ahead

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Quantum Aspects of Space-Time and Matter
Online Talk
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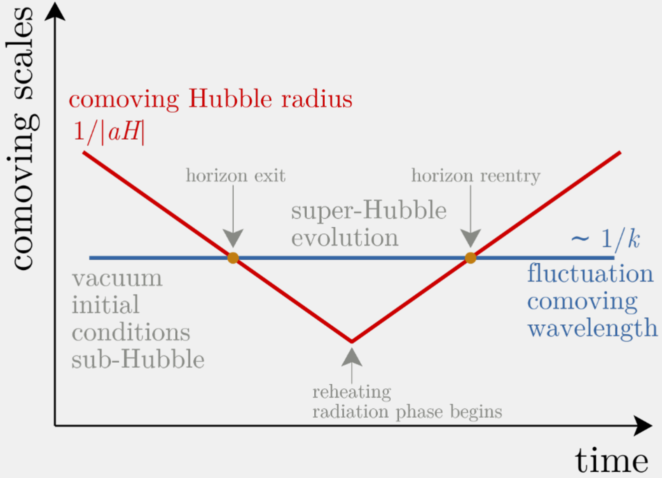
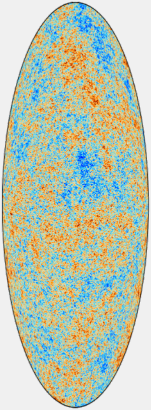
Very early universe cosmology — the standard picture





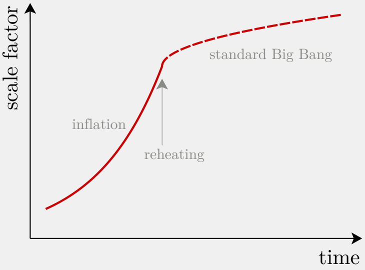
Basic requirements to solve the horizon problem and explain the formation of structures

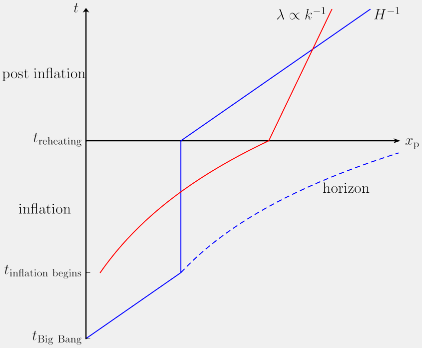
- Suitable initial conditions (e.g., quantum vacuum, thermal state, etc.)
- A sufficiently long phase of evolution over which the comoving Hubble radius shrinks:

$$\frac{\mathrm{d}}{\mathrm{d}t}|aH|^{-1} < 0$$

Example: inflation

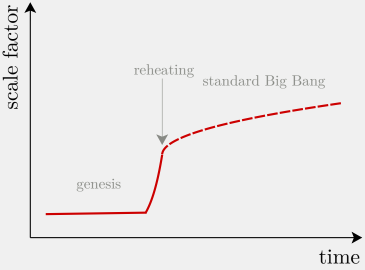
$$a(t) \sim e^{Ht}\,,\ H \approx \text{const.} \implies 1/|aH| \text{ shrinks}$$

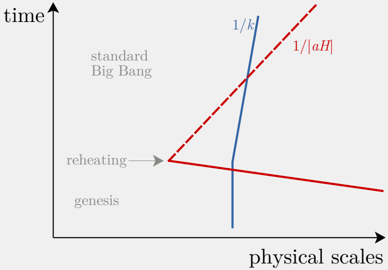




Example: genesis

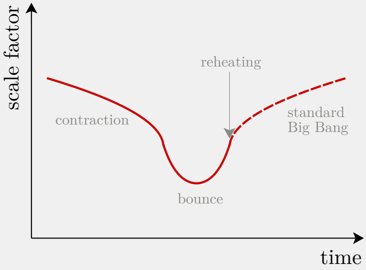
$$a \approx \text{const.}\,, \;\; H \approx 0 \implies 1/|aH| \text{ shrinks}$$

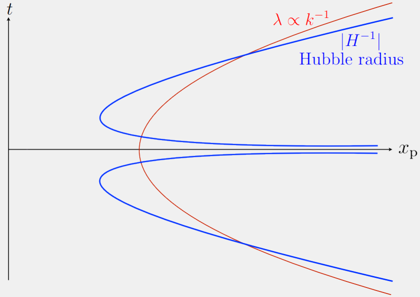




Example: contraction and bounce

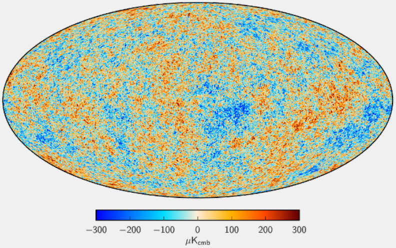
$$a \sim (-t)^{\frac{2}{3(1+w)}}\,,\ w > -1/3\,,\ t < 0\,,\ H < 0 \implies 1/|aH|\ \text{shrinks}$$

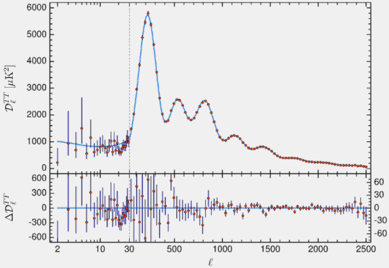




But what do we observe?

Plots from Planck [1502.01582,1807.06209]

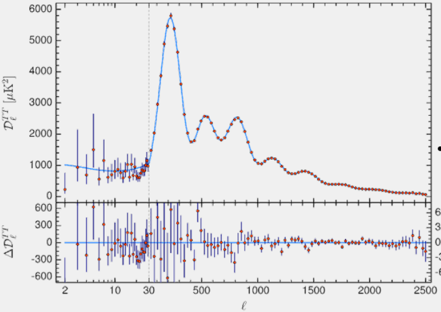


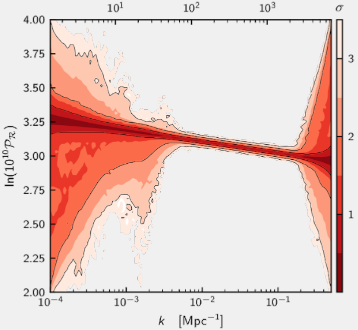


$$\frac{\delta T}{\bar{T}_{\mathrm{cmb}}} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta,\phi)\,, \qquad \mathcal{D}_\ell^{TT} = \frac{\ell(\ell+1)}{2\pi(2\ell+1)} \sum_{m=-\ell}^{\ell} \langle a_{\ell m}^* a_{\ell m} \rangle$$

So what do we know about the very early universe?

Plots and numbers from Planck [1807.06209,1807.06211]





$$\mathcal{P}_\mathcal{R}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 = A_\mathrm{s}\left(\frac{k}{k_\mathrm{pivot}}\right)^{n_\mathrm{s}-1}$$

$$\ln(10^{10}A_\mathrm{s}) = 3.047 \pm 0.014\,, \qquad n_\mathrm{s} = 0.9665 \pm 0.0038$$

$\mathcal{R}$ = curvature perturbations = scalar metric pert. + matter pert. $\supset \delta g_{ij} = \zeta\delta_{ij}$, $\delta\rho$

What do we not see?

Numbers from Planck [1502.01592,1807.06211]

- Running:

$$\alpha_{\mathrm{s}} \equiv \frac{\mathrm{d}n_{\mathrm{s}}}{\mathrm{d}\ln k} = -0.005 \pm 0.013$$

- Non-Gaussianities:

$$\langle(\delta T)^3\rangle \sim \langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle$$

$$f_{\mathrm{NL}}^{\mathrm{local}} = 0.8 \pm 5.0\,, \qquad f_{\mathrm{NL}}^{\mathrm{equil}} = -4 \pm 43\,, \qquad f_{\mathrm{NL}}^{\mathrm{ortho}} = -26 \pm 21$$

- Tensor perturbations:

$$\delta g_{ij} = h_{ij}\,, \quad h^i{}_i = \partial_i h^i{}_j = 0\,, \quad \mathcal{P}_{\mathrm{t}}(k) = \frac{k^3}{2\pi^2}|h_k|^2 = A_{\mathrm{t}}(k/k_\star)^{n_{\mathrm{t}}}$$

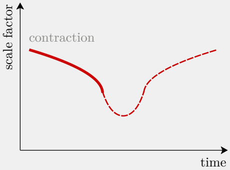
$$r \equiv \frac{\mathcal{P}_{\mathrm{t}}}{\mathcal{P}_{\mathcal{R}}} < 0.07 \quad (95\,\%\,\mathrm{CL})$$

What do the theories predict?

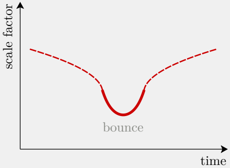
- At face value, a wealth of inflationary models can match the above numbers (some better than others e.g.. Martin et al. [1312.3529])
- Can any of the alternatives do just as well?
  - Example of ‘genesis’ scenario: string gas cosmology Brandenberger & Vafa [89] $\longrightarrow$ can predict many of the numbers, but some work to do on the theoretical foundation e.g., Brandenberger [1105.3247]
  - Bouncing cosmology $\longrightarrow$ the topic of the rest of this talk!

Outline for the rest of this talk

1. Review of **matter bounce cosmology** and **Ekpyrotic cosmology**
   - some models and predictions
   - future developments



2. Review of **non-singular cosmology**;
   or how can a cosmological 'crunching' singularity be avoided?
   - some models and their features
   - future developments



Scale invariance

$$\text{Goal}: \ \mathcal{P}_{\mathcal{R}} \sim k^3|\mathcal{R}_k|^2 \sim A_{\mathrm{s}} k^{n_{\mathrm{s}}-1}\,, \qquad n_{\mathrm{s}} \approx 1$$

- Linear perturbations for GR + matter:

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0 \qquad \text{(Sasaki-Mukhanov eqn.)}$$

$$' = \partial_\tau\,, \quad v_k = z\mathcal{R}_k\,, \quad z^2 = 2\epsilon a^2\,, \quad \epsilon = \frac{3}{2}\left(1 + \frac{p}{\rho}\right)$$

- With $z''/z = 2/\tau^2$ and a quantum vacuum initially ($v_k \to e^{-ik\tau}/\sqrt{2k}$ as $-k\tau \to \infty$), one finds

$$v_k(\tau) \overset{-k\tau\to 0}{\sim} \frac{1}{k^{3/2}\tau} \implies \mathcal{P}_{\mathcal{R}} \sim k^3|k^{-3/2}|^2$$

- If $p/\rho = \mathrm{const.}$, then one needs $z''/z = a''/a = 2/\tau^2$

Duality

Wands [gr-qc/9809062], Finelli & Brandenberger [hep-th/0112249]

$$a(\tau) = a_0(-\tau)^n \implies \frac{a''}{a} = \frac{n(n-1)}{\tau^2} \stackrel{!}{=} \frac{2}{\tau^2} \iff n = -1, 2$$

- This leaves us with two possibilities:
  exponential expansion or **matter-dominated contraction**

$$a(\tau) = \frac{1}{H(-\tau)} \qquad \text{or} \qquad a(\tau) = a_0(-\tau)^2$$
$$\iff a(t) \propto e^{Ht} \qquad \text{or} \qquad a(t) \propto (-t)^{2/3}$$

- The former is inflation. The latter is **matter bounce cosmology**

Successes and problems of matter bounce cosmology

- Easily modeled by a scalar field or dust fluid

- Scale invariant curvature perturbations ✓
- Amplitude given by the scale of the bounce: $A_\mathrm{s} \sim (H_\mathrm{b}/M_\mathrm{pl})^2$ ✓
- $\mathcal{O}(1)$ non-Gaussianities Cai et al. [0903.0631] ✓

- A red tilt $n_\mathrm{s} < 1$, $|n_\mathrm{s} - 1| \ll 1$, and not too much running $\alpha_\mathrm{s} \approx 0$ requires some tuning: $p_\mathrm{eff}/\rho_\mathrm{eff} \approx \mathrm{const.} < 0$ and $|p_\mathrm{eff}/\rho_\mathrm{eff}| \ll 1$

- Scale invariant though large tensor perturbations, i.e., $r \sim \mathcal{O}(10)$ ✗
- Unstable w.r.t. anisotropies ✗

Tensor perturbations in matter bounce cosmology

- Tensor modes:

$$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0\,, \quad u_k = a h_k$$

$\longrightarrow$ same EOM as scalar modes when the equation of state is constant
$\implies$ with same initial conditions, the same amplitude and spectrum follows

- With the proper normalizations one finds $r = 24$!

Possible resolution #1

- What if $c_\mathrm{s} \ll 1$, e.g., with a k-essence scalar field?
- Curvature perturbations are amplified:

$$v_k'' + \left(c_\mathrm{s}^2 k^2 - \frac{2}{\tau^2}\right) v_k = 0 \implies \mathcal{P}_\mathcal{R} \sim \frac{1}{c_\mathrm{s}} \frac{H_\mathrm{b}^2}{M_\mathrm{pl}^2} \implies r = 24 c_\mathrm{s}$$

- $r < 0.07 \iff c_\mathrm{s} \lesssim 0.003$
- But $c_\mathrm{s} \ll 1 \implies$ strong coupling Baumann et al. [1101.3320]
- So the scalar three-point function is also amplified Li, JQ et al. [1612.02036]

$$\text{e.g.,} \qquad f_\mathrm{NL}^\mathrm{local} \simeq -\frac{165}{16} + \frac{65}{8 c_\mathrm{s}^2} \gg 1$$

- Cannot simultaneously satisfy observational bounds on r and f_NL
- Also, $c_\mathrm{s} \ll 1$ with a fluid $\implies$ Jeans (gravitational) instability $\implies$ black hole formation JQ & Brandenberger [1609.02556]

Other possible resolutions

- ✗ If $\mathcal{R}$ grows during the non-singular bouncing phase, r can be suppressed, but again, large non-Gaussianities are created $\implies$ again, observational bounds on r and $f_{\rm NL}$ cannot be simultaneously met JQ et al. [1508.04141]
- ✓ Change the tensor sector with a massive graviton $\implies$ $\mathcal{P}_{\rm t}$ is blue tilted such that $r \ll 0.07$ on observational scales Lin, JQ & Brandenberger [1711.10472]
- ✓ Work with a more general scalar field, e.g., Horndeski with specific functions G_2, G_3, etc. e.g., Akama et al. [1908.10663], Nandi [2003.02066]

The problem of anisotropies

- Consider

$$ds^2 = -\mathrm{d}t^2 + a^2 \sum_{i=1}^{3} e^{2\theta_i} (\mathrm{d}x^i)^2\,, \qquad \sum_{i=1}^{3} \theta_i = 0$$

- Einstein gravity = Friedmann equations + anisotropies:

$$\ddot{\theta}_i + 3H\dot{\theta}_i = 0 \implies \dot{\theta}_i \sim a^{-3} \implies \rho_\theta \sim \sum_{i=1}^{3} \dot{\theta}_i^2 \sim a^{-6}$$

- Analogous to a massless scalar field

$$\mathcal{L}_\theta = -\frac{1}{2}\partial_\mu\theta\partial^\mu\theta \implies p_\theta = \rho_\theta$$

- Anisotropies dominate at high energies:

$$H^2 = -\frac{k}{a^2} + \frac{\Lambda}{3} + \frac{1}{3M_{\mathrm{pl}}^2}\left(\frac{\rho_{\mathrm{m}}^0}{a^3} + \frac{\rho_{\mathrm{rad}}^0}{a^4} + \frac{\rho_\theta^0}{a^6}\right)$$

Ekpyrotic cosmology

- Original proposal comes from string theory, where two $4D$ branes live in $5D$ Khoury et al. [hep-th/0103239], ...
- The distance between the branes is a modulus with potential

$$V(\phi) = -V_0 e^{-c\phi}\,, \quad V_0 > 0\,, \quad c \gg \sqrt{6}$$

- This acts as an attractive force between the two branes, leading to a phase of slow contraction:

$$a(t) \propto (-t)^{2/c^2}\,, \qquad w \equiv \frac{p}{\rho} = \frac{c^2}{3} - 1 \gg 1$$

Perturbations in Ekpyrotic cosmology

- Original model predicts

$$n_\mathrm{s} - 1 = n_\mathrm{t} = \frac{2c^2}{c^2 - 2} \overset{c\to\infty}{\simeq} 2$$

- Latest proposals suggest to add an entropic field as e.g., Fertig et al. [1310.8133]

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{2}e^{-b\phi}\partial_\mu\chi\partial^\mu\chi$$

$$\implies n_\mathrm{s} - 1 \simeq 2\left(1 - \frac{b}{c}\right) \overset{b\approx c}{\approx} 0$$

- $\implies |f_\mathrm{NL}| \sim \mathcal{O}(1 - 10)$ e.g., Fertig et al. [1607.05663]

Successes and problems of Ekpyrotic cosmology

- Easily modeled by a scalar field, motivated by string theory

- Scale invariant curvature perturbations, though for two-field models only $\checkmark$
- $\mathcal{O}(1-10)$ non-Gaussianities $\checkmark$
- Blue tensor power spectrum, so r effectively vanishing on observable scales $\checkmark$

- Usually washes out anisotropies $\checkmark$

Anisotropies revisited

- Ekpyrotic field now dominates at high energies:

$$H^2 = -\frac{k}{a^2} + \frac{\Lambda}{3} + \frac{1}{3M_{\rm pl}^2}\left(\frac{\rho_{\rm m}^0}{a^3} + \frac{\rho_{\rm rad}^0}{a^4} + \frac{\rho_\theta^0}{a^6} + \frac{\rho_{\rm ek}^0}{a^{3(1+w)}}\right)$$

- Numerical studies show that arbitrary initial anisotropies can be 'washed out' in an Ekpyrotic contracting phase Garfinkle et al. [0808.0542]
- But if the Ekpyrotic 'fluid' is also anisotropic, i.e., for $i, j = 1, 2, 3$,

$$p_i = w_i \rho\,, \qquad w_i \gg 1\,, \qquad w_i \neq w_j \ \forall\ i \neq j\,,$$

then anisotropies can be sourced again Barrow & Ganguly [1510.01095]

$$\ddot{\theta}_i + 3H\dot{\theta}_i = \mathcal{S}_i[p_j - \langle p \rangle]$$

What could really tell this apart from inflation?

Cosmological Collider Physics

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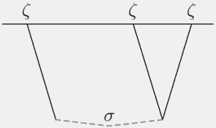
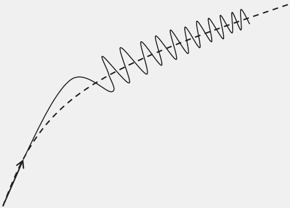


- Heavy fields in inflation leave oscillations in the correlation functions
- E.g., quasi-single field, classically excited or oscillating quantum mechanically

  $\Longrightarrow$ oscillating features in the n-point functions e.g., Chen [1104.1323]



- Same happens for alternatives! (though much less studied)

Oscillations from alternatives

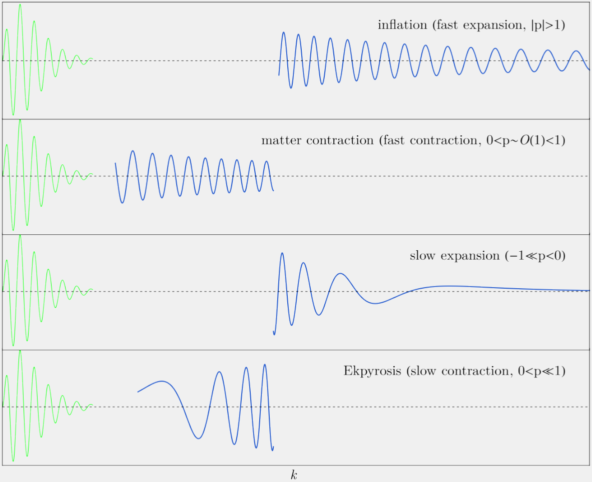
- Massive field fluctuations:

$$\ddot{v}_k + 3H\dot{v}_k + \frac{k^2}{a^2}v_k + m^2 v_k = 0$$
$$\Longrightarrow v_k \sim \exp\left[\pm im\int^{ma/k}\mathrm{d}z\,\sqrt{1+z^{-2}}\frac{\mathrm{d}a^{(-1)}(kz/m)}{\mathrm{d}z}\right]$$

- $a(t) \sim |t|^n \implies a^{(-1)}(t) = a(t)^{1/n}$

$$\longrightarrow \frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}} \sim \sin(k^{1/n})$$

Oscillations from alternatives



e.g., Chen et al. [1411.2349,1809.02603]

Future of alternative models?

- Finding distinctive, observable features
  e.g., computing signal from actual models with massive fields ('pheno') ongoing work
- Black holes may generically form during contraction JQ & Brandenberger [1609.02556]
  Does it leave specific signals? GWs, PBHs, γ-rays? Barrau et al. [1711.05301], Chen et al. [1609.02571], Carr et al. [1104.3796,1402.1437,1701.05750,1704.02919]
- Building concrete UV models (not in the swampland!)
- Developing new scenarios and new approaches:
  - If black holes generically form, could they play a role at high energies, e.g., at the string scale? Veneziano [hep-th/0312182], Mathur [0803.3727], Masoumi [1505.06787], JQ et al. [1809.01658]
  - Alternatively, could the bounce act as a 'filter', where collapsing universes fail, while others (e.g., Ekpyrotic dominated) survive and explain our Universe? e.g., Lehners [1107.4551]
  - Quantum cosmology models...
  - etc.

The bouncing phase: how can we avoid a singularity?

- GR + effective matter satisfying the null energy condition (NEC) $\implies$ singularity singularity theorems by Penrose and Hawking
- $\longrightarrow$ need to violate the NEC, with e.g.:
  - quantum fields
  - modified gravity
  - full quantum gravity
- Why is this not too crazy? E.g.,
  - traversable wormholes Maldacena et al. [1807.04726]
  - 'averaged' energy conditions, e.g. Freivogel & Krommydas [1807.03808]

$$\langle T_{\mu\nu}k^\mu k^\nu\rangle_\tau \geq -\frac{\mathcal{O}(1)}{G_{\mathrm{N}}\tau^2}$$

  - α' corrections in string theory
  - minimal fundamental length in quantum gravity Hossenfelder [1203.6191]
  - etc.

One approach to non-singular cosmology

- Introduce a new, very generic degree of freedom: Horndeski [74] theory

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$+ G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],$$
$$X \equiv -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\,, \qquad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

- Choose the $G_i(\phi, X)$'s in order to violate the NEC for a short period of time e.g., Cai et al. [1206.2382]

- Is the resulting effective theory stable?

Perturbations and (in)stability

- 2nd-order perturbed actions ($\delta\phi = 0$ gauge):

$$S^{(2)}_{\mathrm{tensor}} = \frac{1}{8}\int \mathrm{d}^3x\mathrm{d}t\, a^3\left[\mathcal{G}_T\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla}h_{ij})^2\right]$$

$$\delta g_{ij} = -2a^2\zeta\delta_{ij} \implies S^{(2)}_{\mathrm{scalar}} = \frac{1}{2}\int \mathrm{d}^3x\mathrm{d}t\, a^3\left[\mathcal{G}_S\dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\vec{\nabla}\zeta)^2\right]$$

- Conditions for stability (e.g., scalar sector):

$$\mathcal{G}_S = \mathcal{G}_S[G_i(\phi, X)] > 0 \Leftrightarrow \text{no ghost instability}\,,$$

$$\mathcal{F}_S = \mathcal{F}_S[G_i(\phi, X)] > 0 \Leftrightarrow \text{no gradient instability}$$

No-go theorem

- Within Horndeski theories, it is **not** possible to have a geodesically complete spacetime and be free of both ghost and gradient instabilities at all times Libanov et al. [1605.05992], Kobayashi [1606.05831], ...

$$\mathcal{G}_S(t) > 0\,,\ \mathcal{F}_S(t) > 0\,,\ \mathcal{G}_T(t) > 0\,,\ \mathcal{F}_T(t) > 0\,,\ \forall t \in (-\infty, \infty) \quad ✗$$

- Can also be shown in effective field theory (EFT) Cai et al. [1610.03400,1701.04330], Creminelli et al. [1610.04207]
- The no-go can be evaded only if:
  - In EFT, include higher-order operators e.g., Cai & Piao [1705.03401,1707.01017]
  - Work with beyond-Horndeski theories e.g., Kolevatov et al. [1705.06626]

Limiting curvature

- Different approach to singularity resolution: impose constraint equations that ensure the boundedness of curvature
  $\implies$ **limiting curvature**

- Example of implementation Mukhanov & Brandenberger [92], Brandenberger et al. [gr-qc/9303001], ...

$$S = S_{\mathrm{EH}} + \int \mathrm{d}^4x\,\sqrt{-g}\left[\sum_{i=1}^{n}\varphi_i I_i(\mathbf{Riem},\boldsymbol{g},\boldsymbol{\nabla}) - V(\varphi_1,\dots,\varphi_n)\right]$$

$$\delta_{\varphi_i}S = 0 \implies I_i = \partial_{\varphi_i}V$$

$$|\partial_{\varphi_i}V| < \infty\ \forall\varphi_i \implies \text{bounded curvature}$$

- Concrete model (e.g., $n=2$)

$$I_1 = \sqrt{12R_{\mu\nu}R^{\mu\nu} - 3R^2} \overset{\mathrm{FRW}}{\propto} \dot{H}\,, \qquad I_2 = R + I_1 \overset{\mathrm{FRW}}{\propto} H^2$$

$\longrightarrow$ non-singular background cosmology, but severe instabilities Yoshida, JQ et al. [1704.04184]

- Another implementation of limiting curvature: mimetic gravity Chamseddine & Mukhanov [1308.5410,1612.05860], ...

$$S = S_{\rm EH} + \int \mathrm{d}^4x\,\sqrt{-g}\,[\lambda(\partial_\mu\phi\partial^\mu\phi + 1) + \chi\Box\phi - V(\chi)]$$
$$\delta_\lambda S = 0 \implies \partial_\mu\phi\partial^\mu\phi = -1$$
$$\delta_\chi S = 0 \implies \Box\phi = \partial_\chi V$$

- E.g., $\phi = t \implies \Box\phi = 3H$, so bounding $\partial_\chi V$ ensures H does not blow up

- Yet, mimetic gravity suffers from (gradient) instabilities Ijjas et al. [1604.08586], Firouzjahi et al. [1703.02923], Langlois et al. [1802.03394], ...

Cuscuton gravity

- Setup: GR + non-dynamical scalar field ϕ on cosmological background
- Subclass of 'minimally-modified gravity' (modified gravity with only 2 d.o.f., i.e., the 2 tensor modes of GR) Lin & Mukohyama [1708.03757], Mukohyama & Noui [1905.02000], ...
- Original implementation: start with k-essence theory Afshordi et al. [hep-th/0609150], ...

$$S = S_{\mathrm{EH}} + \int \mathrm{d}^4x \sqrt{-g} P(X,\phi)\,, \qquad X \equiv -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\delta_\phi S = 0 \overset{\mathrm{FRW}}{\Longrightarrow} (P_{,X} + 2XP_{,XX})\ddot{\phi} + 3HP_{,X}\dot{\phi} + P_{,X\phi}\dot{\phi}^2 - P_{,\phi} = 0$$

- Requiring $P_{,X} + 2XP_{,XX} = 0$ sets

$$P(X,\phi) = c_1(\phi)\sqrt{|X|} + c_2(\phi)$$

- Rescaling ϕ, we can write

$$\mathcal{L}_{\mathrm{cuscuton}} = \pm M_L^2\sqrt{2X} - V(\phi)\,, \qquad \partial_\mu\phi \text{ timelike}$$

- EOM becomes a constraint equation:

$$\mp\mathrm{sgn}(\dot{\phi})3M_L^2H=\partial_\phi V$$

$\longrightarrow$ limiting extrinsic curvature

$$M_L^2K=\partial_\phi V\,,\qquad K=\nabla_\mu u^\mu\,,\qquad u_\mu=\pm\frac{\partial_\mu\phi}{\sqrt{2X}}$$

$\longrightarrow$ non-singular bouncing models Boruah et al. [1802.06818]

- Cuscuton fluctuations do not propagate:

$$S^{(2)}_{\mathrm{scalar}}=\int\mathrm{d}^3x\mathrm{d}t\,a^3\left(\mathcal{G}_S\dot{\zeta}^2-\frac{\mathcal{F}_S}{a^2}(\vec{\nabla}\zeta)^2\right)\,,$$

$$\mathcal{G}_S=\frac{X}{H^2}(P_{,X}+2XP_{,XX})=0\,,\qquad\mathcal{F}_S=-M_{\mathrm{pl}}^2\dot{H}/H^2$$

$\longrightarrow$ adding matter, stable curvature perturbations at all times JQ & Yoshida [1911.06040]

- True also for generalizations Iyonaga et al. [1809.10935]

Suggests a new approach

Sakakihara, Yoshida, Takahaski & JQ [2005.xxxxx]

$$S = S_{\mathrm{EH}} + \int \mathrm{d}^3x\mathrm{d}t\, N\sqrt{-\gamma}\left[\sum_{i=1}^{n}\varphi_i I_i(\boldsymbol{K},\boldsymbol{\gamma},\mathbf{D}) - V(\varphi_1,...,\varphi_n)\right]$$

- $K = \nabla^\mu n_\mu$, where $n_\mu = \nabla_\mu\phi$ (mimetic) or $n_\mu = u_\mu$ (cuscuton) such that $n_\mu n^\mu = -1$ (normal, unit vector)
- In FLRW, bound $K \propto H$

- Mimetic gravity $\longrightarrow \mathcal{L} = \frac{R}{2} + \lambda(\partial_\mu\phi\partial^\mu\phi + 1) + \chi\Box\phi - V(\chi)$
- Cuscuton $\longrightarrow \mathcal{L} = \frac{R}{2} + \lambda(u_\mu u^\mu + 1) + \chi\nabla^\mu u_\mu - V(\chi)$
- Cuscuton has one fewer d.o.f. than mimetic theory

- Generalized to a Bianchi universe, bound $K^\mu{}_\nu K^\nu{}_\mu \propto \mathrm{anisotropies}$

Future of singularity resolution?

- Confirming stability beyond the 2nd-order perturbed action
  For standard Horndeski, we run into strong coupling ($c_\mathrm{s} \to 0$) or even non-unitarity e.g., de Rham & Melville [1703.00025], Dobre et al. [1712.10272]
  Beyond-Horndeski models seem to be doing better e.g., Mironov et al. [1910.07019]
  How about cuscuton models? ongoing
  How about stability non-perturbatively e.g., Ijjas et al. [1809.07010]
- UV completion?
  String theory realizations?
  E.g., non-perturbative solutions in α' ongoing with Bernardo, Franzmann & Lehners

Conclusions

- Matter bounce cosmology $\longrightarrow$ nice idea, but perhaps not on the best footing at this point
- Ekpyrotic cosmology $\longrightarrow$ works nicely
- Need to put them to the test even more $\longrightarrow$ massive fields
- It would be neat to find yet more ideas

- Hard to find non-singular cosmology free of instabilities
- Possible with higher-order operators, with many free functions
- Or with constrained system, where the new d.o.f. disappears in cosmology
- Many questions remain to address to make those viable theories at high energies for the very early universe

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